CONSUMER WELFARE AND VARIETY

By

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Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

Consumer Welfare and Variety

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The objective of this dissertation is to show the importance of allowing the number of varieties to change in the economic welfare analyses. It is a well-accepted idea that as the variety of goods and services available increases, consumers' utilities increase. However, detailed studies are scarce.

The first two essays focus on how international trade affects the welfare of small countries by changing the number of varieties of goods available. Previous studies have measured the loss from imposing tariffs as being very small and, hence, concluded that closing the economy from the rest of the world could stimulate economic growth in a small country. This led many developing countries to adopt import substitution policies, which, unfortunately, created an even bigger gap between them and developed countries. This conflict between theory and reality should be analyzed. Both essays feature a model for a developing country's import market that assumes foreign firms must pay a fixed entry cost that differs among firms. Each firm sells a differentiated product. The first essay studies the instantaneous case, while the second looks at an infinite horizon model.

The results show that a nation's welfare is always improved whenever its tariff rate is reduced because more firms enter and additional varieties are available to consumers. Furthermore, simulation results show that the welfare gain can be large.

The third essay analyzes the effect of variety on apartment rents. An empirical study shows there is an increase in rent during the first fifteen years despite the depreciation of the building. Simulation results suggest that this rent hike is too rapid to be fully explained by the conventional theory, that the growth of the city's border causes the rent increase. Thus, the effect of variety change, shown to be significant in the preceding chapters, is introduced. An individual's utility increases as the immediate surrounding of his/her living place, an "activity circle," expands and offers more varieties of daily activities. This study provides the first empirical evidence that rent changes depend on both the city's growth rate and the change in the number of varieties available within the activity circle.

CHAPTER 1
INTRODUCTION

It is well known that people benefit from having more choices. Indeed, this fact is the reason monopolistic competitive firms have more market power than perfectly competitive firms: monopolistically firms acquire market power because they can sell their uniqueness to consumers who are willing to pay more for their preferences to be better met. There is almost no disagreement on the point that variety benefits consumers, but detailed studies are surprisingly scarce.

This dissertation emphasizes the role played by the number of varieties by studying this issue in two rapidly growing fields of economics, international trade and urban economics. Because variety is assumed to add extra utility, ignoring the effect of variety changes might cause some bias in welfare analyses, which is what I find.

The next two chapters of this dissertation focus how varieties introduced by international trade affect small, less developed countries. Previous studies have ignored the fact that by closing a country from international trade, the nation forgoes the opportunity to introduce varieties of commodities developed outside of the country. Resulting empirical studies report a loss from restricting trade by imposing tariffs as small as one percent of the nation's GDP, and thus concluded that closing the economy, especially from developed countries, could stimulate domestic economic growth. This conclusion has led many LDCs to employ import substitution policies, and brought little or even negative growth to those nations.

The analytical models introduced in the next two chapters have two key assumptions. First, the importing nation is small and less developed, so it cannot produce close substitutes to imported consumer goods, such as automobiles and computers. Second, both models assume that foreign providers of these goods face different levels of entry costs. When the tariff rate is decreased, firms with higher entry costs who had not previously been in the market enter and introduce new goods. Some of the new firms then will not withdraw from the market if the government raises the tariff rate back to its original level because they have already paid the entry costs. This entry cost, a sunk cost, creates hysteresis in the equilibrium number of firms and, hence, the number of varieties of imports.

The second chapter introduces a static model to analyze these effects. It assumes that as the number of varieties grows, the elasticity of demand for a particular product increases. This allows an explicit analysis of the effects on consumer welfare from changing the elasticity of demand. In my model, lowering the tariff rate benefits the economy in two ways: by increasing the variety of imports and by raising the quantity consumed due to the elasticity change. Within this framework, a reduction in the tariff rate always raises the welfare of the country. This discussion strongly recommends that developing countries open their economy by eliminating tariffs. The chapter also provides some empirical evidence on the existence of hysteresis in the foreign suppliers' behavior.

The model introduced in the second chapter investigates the instantaneous effect of a tariff reduction on the nation's welfare. Missing from that model, and from other studies, is the longer-run effect on the nation's welfare from including hysteresis in the number of firms in the market. The third chapter develops an infinite-horizon model that features the hysteresis effect: a reduction in the tariff rate changes the equilibrium number of varieties of products imported to the nation by a higher magnitude than does a rise in the tariff rate by an equal magnitude. This asymmetric movement results in another hysteresis, namely in the nation's welfare. In this analysis, a scenario for the government of a developing country is provided: it cuts the tariff rate once and then (unexpectedly) raises it back to its original level. The resulting magnitude of the change in welfare is surprising. Unlike conventional models that predict small welfare changes, the gain in the welfare can be as much as fifty percent of the original welfare level.

The fourth chapter switches to an urban economics setting and examines the effect changes in the number of varieties of daily activities available in the immediate surrounding area have on an individual's willingness to pay to live in the location. The extra utility created by having more activity varieties must be reflected in the rent the resident pays to live at the location. The objective of this chapter is to show that the standard urban economic theory for how rent changes over time, which lacks consideration of changes in the local number of varieties, underestimates the rate of change in apartment rents as the apartment ages.

The chapter starts by verifying the standard theory qualitatively while showing it is not sufficient quantitatively. Capozza and Helsey (1989, 1990) introduce a model that connects the rate of increase in rent to the rate of city growth. This standard urban economic model has been accepted widely, but little empirical tests have been conducted of it. By comparing a hedonic price function for the apartment rent in fast growing counties in Florida to an index for slower growing counties, it is found that the apartment rent rises more quickly in cities experiencing rapid growth. Thus, this chapter provides the first empirical evidence that there is indeed a positive relationship between the rate of rent increase and the rate of growth of the city in which the apartment is located. However, simulation results suggest that the expansion of the city is not the only factor that explains changes in the rent. Hence, the chapter introduces a new concept of a resident's "activity circle." That is, an individual has his/her circle in which his/her daily activities, such as grocery shopping and dining out, take place. Individuals prefer to have more varieties in this circle (it is better to choose from five different restaurants than only one), so they are willing to pay a higher price to live in a place that offers more variety. Adding this effect onto the model introduced by Capozza and Helsley provides a better fitting explanation for the empirical results on the change in rent over time.

The results from those three chapters clearly point out the significance of variety change in the consumers' welfare or utility. The underestimation of the effect of the government's trade policy on the nation's welfare can cause misguided policies an the national level. The underestimation of the growth rate of the apartment rents can cause misguided urban development plans. These are just a few examples. If this important concept is overlooked in welfare or cost-benefit analysis in any fields of economic, underestimation of the change in welfare might occur.

CHAPTER 2
TARIFFS AND HYSTERESIS: THEORY AND EMPIRICS

Introduction

For the past decades, many trade economists have been trying to estimate the loss of welfare caused by trade restrictions. Harberger (1959) estimated the loss as up to 2.5% of the nation's income using the Chilean economy as his example. Feenstra (1992), by analyzing import demand and export supply curves, estimated the loss of U.S. gross domestic product due to the restrictions as less than one per cent. Krugman and Obstfeld (1994) list estimations of the loss for different nations: according to the list, Brazil's loss in 1966 was 9.5 per cent of its national income, Pakistan lost 6.2 per cent in 1963, Mexico's loss was 2.5 per cent in 1960, and the United States lost 0.26 per cent in 1983.

Is the distortion caused by trade restrictions that small? After some Asian nations' economic success, apparently caused by opening their economy, and the collapse of some Latin American countries that restricted trade, some economists started to question the low estimates of trade distortions. Then the idea of "new goods" appeared. In particular, when a nation closes its economy, it loses the opportunity to introduce newly developed or invented goods from rest of the world. As a result, if it is a final good, consumers cannot consume that good, so no surplus from the good is created. If it is an intermediate good, newly developed to increase productivity (for instance, a new production technology), then the economy suffers from a less efficient production system and lower national output than it could otherwise have enjoyed.

Romer (1994) studied this latter case. He showed that if a final good is produced with labor and imported capital inputs, when the number of varieties of imported capital is allowed to change, a tariff imposed on capital goods will reduce the national output level by twice as much as the tariff rate. In his model, each entrepreneur providing a different capital good has a different level of fixed cost of entry which cannot be collected at the time of withdrawal. A firm enters the market if it sees profitability net of this sunk cost. The firm with the smallest fixed cost enters first, then as profitability increases, possibly because of reductions in the tariff rate, firms with higher fixed costs start to introduce their products.

Most economists agree that when consumers face more choices or more varieties to choose from, their elasticity of demand for any particular commodity will increase. There are, however, few studies that explicitly show the effect this elasticity shift has on welfare. The relationship between new goods introduced to a country from the outside and the nation's welfare plays a more important role in developing countries than in developed countries. In an industrialized country, innovation of new goods and production of old goods automatically occurs if the nation has a large enough supply of labor[1]. As all North-South models assume, a developing country is less likely to be an innovating country. For this reason, the interest of this paper will be focused on developing countries, where new varieties are made available through imports.

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[1] If the North does not have satisfactory large labor force, trade restrictions will slow the growth. See Krugman (1979) and Segerstrom et. al. (1990) for more detail.

Significance of the sunk cost of entry brings up the idea of hysteresis in suppliers' behavior. Hysteresis is a term used when an external shock changes a variable but the variable fails to return to its original level after the source of the shock is removed. Sunk cost creates a gap between the profit level at which a firm will enter and the profit level at which the firm will exit. This gap makes for hysteresis in supplier's behavior.

Section one performs a cost-benefit analysis of a tariff when firms have different levels of sunk costs that must be paid to enter and consumers' elasticity of demand depends on the number of varieties available to them. The result is quite surprising; it turns out that the nation's domestic welfare will always be reduced when the government rises the tariff rate regardless of the level of elasticity of demand or initial tariff rate. Section two provides empirical evidence to justify the fundamental and vital assumption made in section one, namely, the significance of sunk cost in the entrepreneur's decision-making process by testing for hysteresis in the behavior of export suppliers.

Cost-Benefit Analysis

Romer (1994) showed the loss from a tariff imposed on imported intermediate goods in terms of the nation's gross national product. Unfortunately, his analysis is limited to the production side of the economy. Of course, a nation's output level has a significant influence on worker's income, and thus welfare. The purpose of this section is to analyze the direct effect of a tariff on welfare through changes in consumer surplus and tariff revenue.

Let the inverse demand function for an imported good be $P_i(x_i)$ which is given by the following function

$$P_i(x_i) = x_i^{-\frac{\alpha}{N}} \tag{2-1}$$

where α is a positive constant and N is the total number of varieties of goods imported. Assume N is larger than α. Hence, the own-price elasticity of demand is

$$\varepsilon_i = -\frac{dx_i}{dP_i} = \frac{N}{\alpha} > 0\,. \tag{2-2}$$

Notice that the elasticity is a constant for a given number of varieties but becomes larger as more goods enter the market. The more varieties become available, the more consumers have choices, and the more price sensitive the demand will be.

Turn now to the producer side. Under some types of imperfect competition (in this case, monopoly), a firm producing good *i* can be denoted as the firm *i*, as in Romer (1994). Firm *i* has a constant marginal cost, c_i, and a fixed cost of introducing a new good, FC(*i*). Following Romer again, we assume $FC(i)=\mu i$ where μ is a constant. This assumption gives different levels of fixed costs to different firms. The introduction cost is interpreted as a cost paid for advertising or setting up a marketing network, so it is a sunk cost once the firm enters. For simplicity, assume symmetry across firms and demands for foreign goods except the fixed costs, so that $x_i=x$, $P_i=P$, $\varepsilon_i=\varepsilon$ and $c_i=c$.

The objective of a foreign firm which has already entered the market is to maximize its profit, given by

$$(1-\tau)P(x)x - cx, \tag{2-3}$$

where τ is an ad valorem tariff rate. Each firm takes N and τ given. Then the first order condition for profit maximization problem is

$$(1-\tau)[P(x) + P'(x)x] = c. \tag{2-4}$$

Substituting the inverse demand function defined earlier into equation (2-4), the equilibrium quantity each firm produces is

$$x^*(\tau, N) = \left[\left(1 - \frac{\alpha}{N}\right)(1-\tau)/c\right]^{\frac{N}{\alpha}} = \left[\left(1 + \frac{1}{\varepsilon}\right)(1-\tau)/c\right]^{\frac{N}{\alpha}}. \tag{2-5}$$

This is the usual expression for a firm with some market power with the mark-up for the firm represented by the first term in the square brackets. We see from the exponent that the higher the elasticity of demand, the lower is the firm's market power.

A new firm will enter if it sees profitability in the market net of its fixed cost, that is if

$$(1-\tau)P(x)x - cx \geq FC(i) = \mu i, \tag{2-6}$$

for the *i*th firm. Because FC(*i*) is an increasing function of *i*, the last firm to enter, say the Nth, earns zero profit after it pays the introduction cost. The equilibrium number of varieties with a given tariff rate, thus, takes the form of

$$N^*(\tau) = \frac{1}{\mu}\left[(1-\tau)P(x^*(\tau)) - c\right]x^*(\tau). \tag{2-7}$$

Here, the integer constraint is ignored for calculation purpose. A function for the common marginal cost can be derived from the first order condition of the profit maximization problem.

$$c = (1-\tau)\left(1 - \frac{\alpha}{N^*(\tau)}\right)x^*(\tau)^{-\frac{\alpha}{N^*}}. \tag{2-8}$$

Substituting the above function for the marginal cost and the equilibrium values of P and x, equation (2-7) becomes

$$N^*(\tau) = \frac{\alpha}{\mu(N^*(\tau)-\alpha)} c x^*(\tau)$$

$$= \frac{\alpha}{\mu(N^*-\alpha)} c^{1-\frac{N^*}{\alpha}} \left(1-\frac{\alpha}{N^*}\right)^{\frac{N^*}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}} . \tag{2-9}$$

which remains an implicit function for $N^*(\tau)$ because it is impossible to eliminate N^* from the right hand side of the equation.

The most significant implication of the new trade theory is that if a country reduces its tariff rate, consumers will benefit not only from the price reduction of (already) imported goods, but also from increased varieties of goods available to them. The latter effect can be analyzed by differentiating equation (2-9) with respect to tariff rate. Normalizing the common constant marginal cost to be one, (2-9) becomes

$$\begin{aligned} \frac{dN^*(\tau)}{d\tau} = \frac{\alpha}{\mu} \Bigg\{ & -\frac{1}{(N^*-\alpha)^2} \left(1-\frac{\alpha}{N^*}\right)^{\frac{N^*}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}} \\ & + \frac{1}{N^*-\alpha} \left[\frac{1}{\alpha} \ln\left(1-\frac{\alpha}{N^*}\right) + \frac{1}{N^*-\alpha}\right] \left(1-\frac{\alpha}{N^*}\right)^{\frac{N}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}} \\ & + \frac{1}{\alpha(N^*-\alpha)} \left(1-\frac{\alpha}{N^*}\right)^{\frac{N^*}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}} \ln(1-\tau) \Bigg\} \frac{dN^*(\tau)}{d\tau} \\ & - \frac{N^*}{\mu(N^*-\alpha)} \left(1-\frac{\alpha}{N^*}\right)^{\frac{N^*}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}-1} . \end{aligned} \tag{2-10}$$

Rearranging gives

$$\frac{dN^*}{d\tau} = -N^*\left(1-\frac{\alpha}{N^*}\right)^{\frac{N^*}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}-1}$$

$$*\left\{\mu\left(N^{*}-\alpha\right)-\left(1-\frac{\alpha}{N^{*}}\right)^{\frac{N^{*}}{\alpha}}(1-\tau)^{\frac{N^{*}}{\alpha}}\left[\ln\left(1-\frac{\alpha}{N^{*}}\right)+\ln(1-\tau)\right]\right\}^{-1}. \qquad (2\text{-}11)$$

The right hand side of the above expression[2] is always negative, so the total number of varieties imported is negatively related to the tariff rate. As the tariff is lowered, the profit after entry (given by equation [2-3]) rises allowing some firms with slightly higher fixed costs to introduce their products. The last line in equation (2-10) is the direct effect[3] of the tariff and the first three lines are indirect effects. Tariffs will affect the profit by changing the quantity demanded, the direct effect, and by changing the elasticity of demand, causing a shift in the demand, the indirect effect.

A reduction in the tariff will change the domestic consumer surplus associated with a currently marketed good by both shifting the supply curve down and making the demand curve more elastic by bringing in new goods. These effects can be analyzed mathematically by differentiating the consumer surplus at the equilibrium defined as

$$CS=\int_{0}^{x^{*}} x(\tau)^{-\frac{\alpha}{N}}dx-P(x^{*}(\tau))x^{*}(\tau), \qquad (2\text{-}12)$$

with respect to the tariff:

$$\frac{dCS}{d\tau}=\frac{d\left(\int_{0}^{x^{*}} x^{-\frac{\alpha}{N}}dx\right)}{d\tau}-\frac{d\left[P(x^{*})x^{*}(\tau)\right]}{d\tau}$$

---

[2] This does not reflect hysteresis in firms' behavior. Hysteresis means the magnitude should differ when $d\tau>0$ from when $d\tau<0$. As we will see later, however, our interest is not in the magnitude, but the sign of this equation. Adapting hysteresis in the mathematics will complicate the analysis unnecessarily without giving much difference in the result, so it is ignored here.

$$= x^*(\tau)^{-\frac{\alpha}{N^*}} \frac{dx^*}{d\tau} - \left(\frac{\alpha}{N^{*2}} \ln(x^*) \frac{dN^*}{d\tau} + \frac{N^* - \alpha}{N^* x^*} \right) x^*(\tau)^{1-\frac{\alpha}{N^*}} \frac{dx^*}{d\tau}$$

$$= \frac{\alpha}{N^*} x^*(\tau)^{-\frac{\alpha}{N^*}} \left(1 - \frac{x^*}{N^*} \ln(x^*) \frac{dN^*}{dx^*} \right) \frac{dx^*}{d\tau}. \tag{2-13}$$

where

$$\frac{dx^*(\tau)}{d\tau} = \frac{dx^*}{dN^*} \frac{dN^*(\tau)}{d\tau} - \frac{N^*}{\alpha} \left(1 - \frac{\alpha}{N^*} \right)^{\frac{N^*}{\alpha}} (1-\tau)^{\frac{N^*}{\alpha}-1}. \tag{2-14}$$

To determine the sign of equations (2-13) and (2-14), we must know dx^*/dN^*. By totally differentiating the first line of equation (2-9), we obtain

$$dN^* = -\frac{\alpha}{\mu \left(N^* - \alpha \right)^2} x^* dN^* + \frac{\alpha}{\mu \left(N^* - \alpha \right)} dx^*. \tag{2-15}$$

Rearranging the order gives

$$\frac{dx^*}{dN^*} = \frac{\mu}{\alpha} \left(N^* - \alpha \right) + \frac{x^*}{N^* - \alpha}. \tag{2-16}$$

which is positive. This sign occurs because even though introducing a new variety will shift the demand curve for an existing good inward, the higher elasticity makes the curve flatter, causing the new demand curve to intersect the new supply curve at a point south-east of the old equilibrium.

---

[3] If $P(x) = x^{-\alpha}$, then $N^*(\tau) = \frac{\alpha}{\mu(1-\alpha)} (1-\alpha)^{\frac{1}{\alpha}} (1-\tau)^{\frac{1}{\alpha}}$, and $\frac{dN^*(\tau)}{d\tau} = -\frac{1}{\mu(1-\alpha)} (1-\alpha)^{\frac{1}{\alpha}} \left(1-\tau\right)^{\frac{1}{\alpha}-1}$.

With dx^*/dN^* positive and $dN^*/d\tau$ negative, $dx^*/d\tau$ is negative[4]. That shows the consumer surplus associated with an existing variety is negatively related to the tariff rate. For simplicity, all values for x and N hereafter are assumed to be at the equilibrium, so the superscript of a star (*) is omitted. Notice that by substituting equation (2-14), equation (2-13) can also be written as

$$\frac{dCS}{d\tau} = -\frac{1}{1-\tau} x^{1-\frac{\alpha}{N}}$$

$$+\frac{\alpha}{N} x^{1-\frac{\alpha}{N}} \left(\frac{1}{x} \frac{dx}{dn} \frac{dN}{d\tau} - \frac{1}{N} \ln x \frac{dN}{d\tau} + \frac{x}{\alpha(1-\tau)} \ln x \frac{dN}{dx} \right). \qquad (2\text{-}17)$$

Then the second line is the additional effect on the consumer surplus by allowing the elasticity of demand to change and to shift the demand curve. The first term is the conventional "Harberger triangle." This is the change in the consumer surplus due to a movement along the demand curve[5]. Because the second term is always negative, it is clear that fixing the number of varieties (firms) will underestimate the distortion caused by tariff.

Now that the effect of a tariff on the consumer surplus from existing goods has been analyzed, the next step is to examine the total effect on the consumer surplus from all imported goods. This includes the goods provided by the foreign monopolists on the margin of entering to or exiting from the market. Define TCS as the total consumer surplus, which is equal to N(τ)CS due to symmetry.

---

[4] Notice that $\ln(x^*) < 0$ because both $\left(1 - \frac{\alpha}{N^*}\right)$ and $(1-\tau)$ are less than one. See equation (1-5).

$$\frac{dTCS}{d\tau} = CS\frac{dN}{d\tau} + N(\tau)\frac{dCS}{d\tau}, \tag{2-18}$$

where both dN(τ)/dτ and dCS/dτ are negative from the previous analysis. Thus, the tariff and the total consumer surplus associated with imported goods are negatively related. If the tariff is reduced, it will attract new monopolists to enter the market and a new market for a new commodity is created. Consumers receive zero surplus when goods are not available to them, but new goods introduced following a tariff reduction will provide some positive surplus. The first term in equation (2-18) captures this effect. Another element of the welfare effects from tariffs is the government's tariff revenue. The tariff revenue from one particular commodity is given by $TR = \tau P(x)x = \tau x(\tau)^{1-\frac{\alpha}{N}}$.

(2-19)

The effect of a change in tariff on the tariff revenue from an existing commodity can be examined by differentiating equation (2-19) with respect to τ.

$$\begin{aligned}\frac{dTR}{d\tau} &= x(\tau)^{1-\frac{\alpha}{N}} + \tau\left(\frac{\alpha}{N^2}\ln x\frac{dN}{dx} + \frac{N-\alpha}{Nx}\right)x(\tau)^{1-\frac{\alpha}{N}}\frac{dx}{d\tau} \\ &= \frac{N}{\alpha(1-\tau)}x^{-\frac{\alpha}{N}}\left(\frac{\alpha}{N} - \tau\right) \\ &\quad + \frac{\tau}{N}x^{1-\frac{\alpha}{N}}\left(\frac{\alpha}{N}\ln x\frac{dN}{d\tau} - \frac{N-\alpha}{x}\frac{dx}{dN}\frac{dN}{d\tau} - \frac{x}{1-\tau}\ln x\frac{dN}{dx}\right).\end{aligned} \tag{2-20}$$

---

[5] If P(x) = x$^{-\alpha}$, then $\frac{dCS}{d\tau} = -\frac{1}{1-\tau}x^{1-\alpha}$.

The first term in the second line, showing the direct effect of the tariff, is positive only if τ is less than α/N[6]. That means a rise in tariff will increase revenue only when the demand is not very elastic. Another effect is always positively related to the tariff change. An increase in the tariff rate will drive some firms away, fewer varieties makes consumer's demand less elastic, and this causes the new equilibrium quantity higher than it would be otherwise even with the price increase due to a higher tariff rate. That brings more tariff revenue. However, even with a positive indirect effect, if the direct effect is negative, it is possible to have the benefit smaller or even disappear. The range of α and τ which will make the net benefit positive is impossible to determine due to the complicated functional form of $dN/d\tau$.

The effect on the total tariff revenue, TTR, is

$$\frac{dTTR}{d\tau} = \frac{dN^*}{d\tau} TR + N^* \frac{dTR}{d\tau}. \tag{2-21}$$

The second term shows the total effect on all existing commodities whereas the first term shows the effects from changing the number of goods. A rise in the tariff rate will turn away marginal firms, reducing the sources of revenue. This should be negatively related to the tariff rate. Even if tariff revenue from each commodity is positively related to the tariff rate, the second term of the last line of equation (2-21) is negative and so may offset the initial positive effect.

---

[6] If $P(x) = x^{-\alpha}$, then $\frac{dTR}{d\tau} = \frac{1}{\alpha(1-\tau)} x^{1-\alpha}(\alpha - \tau)$, which is positive only if $\tau < \alpha$. Notice the elasticity of demand for this demand function is $1/\alpha$.

Finally, putting this analysis together, it is possible to perform a cost-benefit analysis of a tariff in the import market. This nation's welfare from imported goods is given by

$$
\begin{aligned}
W_I &= TCS + TTR \\
&= N(\tau)CS + N(\tau)TR. \qquad (2\text{-}22)
\end{aligned}
$$

The effect can be examined by differentiating W_I with respect to τ.

$$\frac{dW_I}{d\tau} = (CS + TR)\frac{dN}{d\tau} + N(\tau)\left(\frac{dCS}{d\tau} + \frac{dTR}{d\tau}\right). \qquad (2\text{-}23)$$

From the previous analysis, it is known that $\frac{dN}{d\tau}$ is always negative. Therefore, to determine the sign of equation (2-23), analysis should be focused on the second term. By substituting equations (2-13) and (2-20), the sum of the two derivatives in equation (2-23) becomes

$$
\begin{aligned}
\frac{dCS}{d\tau} + \frac{dTR}{d\tau} &= x^{1-\frac{\alpha}{N}} + x^{-\frac{\alpha}{N}}\frac{dx}{d\tau} + (\tau - 1)x^{1-\frac{\alpha}{N}}\left(\frac{\alpha}{N^2}\ln x\frac{dN}{dx} + \frac{N-\alpha}{Nx}\right)\frac{dx}{x\tau} \\
&= x^{1-\frac{\alpha}{N}} + x^{1-\frac{\alpha}{N}}\left\{\frac{\alpha(\tau-1)}{N^2}\ln x\frac{dN}{dx} + \frac{1}{Nx}[N\tau + \alpha(1-\tau)]\right\}\frac{dx}{d\tau} \\
&= x^{1-\frac{\alpha}{N}}\left[1 + \frac{1}{x}\left(1 + \frac{\tau(N-\alpha)}{Nx}\right)\frac{dx}{d\tau}\right] + \frac{\alpha(\tau-1)}{N^2}x^{1-\frac{\alpha}{N}}\ln x\frac{dN}{dx}\frac{dx}{d\tau}. \qquad (2\text{-}24)
\end{aligned}
$$

Substitute $\frac{dx}{d\tau}$ with equation (2-14), it becomes

$$\frac{dCS}{d\tau}+\frac{dTR}{d\tau}=\frac{x^{1-\frac{\alpha}{N}}}{\alpha(1-\tau)}(\alpha-N-\tau N)$$

$$+x^{-\frac{\alpha}{N}}\left[1+\frac{\tau(N-\alpha)}{N}+\frac{\alpha(\tau-1)x}{N^2}\ln x\right]\frac{dx}{dN}\frac{dN}{d\tau} \qquad (2\text{-}25)$$

This is always negative. Thus, even if a rise in tariff brings a positive benefit from higher tariff revenue, the loss in the consumer surplus always offsets it and the total welfare change becomes negative. A tariff reduction decreases the tariff revenue from each unit of imports, but since the total number of units of each variety imported increases, the total tariff revenue from an existing variety may or may not be reduced. However, the increased units of consumption raises the consumer surplus gained from each variety. The lower tariff also attracts new foreign firms to enter the domestic market. The increased number of varieties raises the elasticity of demand and makes the quantity demanded even higher[7]. The new varieties provide additional sources of consumer surplus and tariff revenue which improve the welfare of the domestic economy. In total, even if the tariff revenue from each variety may be lower, the domestic economy enjoys higher welfare with a lower tariff rate. Within this framework, a reduction in the tariff rate will always make the economy better off without any restrictions on the initial level of tariff. If the tariff structure is uniform, there will be no positive tariff rate that will give the economy a higher level of welfare than free trade. There is no positive optimum uniform tariff. When a higher tariff drives some firms away, the consumer surplus from their commodities disappears, and the loss is too large to be ignored.

Significance of the Sunk Cost : Empirical Evidence

The fundamental assumption in the cost-benefit analysis of the effect of a tariff change in the previous section is that the sunk costs associated with entering a foreign market is a significant factor in an entrepreneur's decision-making process. If it is not, otherwise symmetric firms would simultaneously respond to a change in tariff rate and the market will contain either all varieties in the world or nothing at all. Under that circumstance, revising the tariff rate will not affect the number of firms in the economy unless it moves from below to above or from above to below the prohibitive level.

The empirical model given in this section does not test the significance of a sunk cost to the decision-making process directly due to data availability. Instead, it studies this issue indirectly: A firm will enter the market if it can earn more than it invests. This entry condition is given by

$$\Pi(P,x) \geq FC \;, \tag{2-26}$$

where $\Pi(\cdot)$ is a monopolistic profit a firm in the market earns. This is a general form of equation (2-3). The condition for a firm to stay in the market is

$$\Pi(P,x) \geq 0 \,, \tag{2-27}$$

which is a general form of equation (2-6). This means an operating firm may stay in the market even when the price decreases to below the entry level. In other words, the movement of the import price will have a different impact on the overall export supply to

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[7] Notice that as the elasticity of demand increases, the effect of an elasticity change on the quantity demanded becomes larger ($d^2x/dN^2=2\mu/\alpha>0$). If the elasticity of demand does not depend on the number of varieties available ($\varepsilon=1/\alpha$), dx/dN is a constant.

the country when it increases and attracts new firms to the market, than when it decreases because not as many firms will leave the market as would if the conditions to enter and to stay were the same. With that asymmetric effect, the export supply to a country will be given by

$$\ln Q_t^s = \alpha^s + \beta_1^s t + \beta_2^s \ln\left(\frac{P_t^I}{P_t^F}\right) + \beta_3^s \ln\left(\frac{P_t^I}{P_t^F}\right)_{RAT} . \tag{2-28}$$

where Q_t^s is the total quantity exported to the Home Country (Colombia , Sri Lanka, and Philippines are chosen in this empirical study) from the rest of the world, P_t^I is the price foreign firms receive by exporting to the Home Country net of the tariff, P_t^F is the price the firm faces if it sells the product in its own country, say, the Foreign Country (e.g. for a U.S. firm, it will be the price in the U.S.) and $(P_t^I / P_t^F)_{RAT}$ is a ratchet variable for the relative price. $(P_s^I / P_s^F)_{RAT}$ takes the value of the highest (P_t^I / P_t^F) in the period between t=0 and t=s. The price ratio shows in which market, the Home or the Foreign, the firm can sell its product at higher price. The term with the ratchet will catch the asymmetric responses of firms, which creates hysteresis in their supplying behavior. As the price of its product rises in the Home Country relative to that in the Foreign Country, the firm will export more, so the price ratio should be positively related to the quantity of exports supplied. Thus, the coefficients of both terms are expected to be positive.

Clearly, the import price is an endogenous variable in this export/import market. In order to find appropriate instrumental variables, the obvious place to look is the other side of the market: The demand for imports, specified as

$$\ln Q_t^d = \alpha^d + \beta_1^d t + \beta_2^d \ln\left(\frac{P_t^I\left(1+\tau_t\right)}{P_t^D}\right) + \beta_3^d \ln GDP_t \ . \qquad (2\text{-}29)$$

Here, P_t^D is the price of domestic good in the Home Country, GDP_t is the per capita real gross domestic product of the Home Country, and τ_t is the tariff rate imposed by the government of the Home Country. Demand is inversely related to the relative price of imports, so β_2^d should be negative. The possible instruments, hence, are $\ln GDP_t$, and $\ln(1+\tau_t)$. Furthermore, the two prices in the price ratio variable must be in the same currency unit, so the natural log of exchange rate, $\ln e_t$, can be another instrument. However, the data for the tariff rates for any of the three countries used here as samples is not available. The instruments, therefore, employed here are a constant, $\ln e_t$, trend, $\ln GDP_t$, and $\ln P_t^F$.

For Colombia, the sample period starts from 1972. The reasons for this starting date are in 1972: i) a new government established a new development plan[8], including new exchange rate policy and abandonment of the import substitution policy; and ii) the difference between the official and black market exchange rates became less than 10 per cent[9]. For Sri Lanka, the sample period starts from 1978. This is because between 1970 and 1977, a dual exchange rate system was employed in this country. This policy was abolished in 1977, and a single free-float exchange rate system has been in action since then. Also 1977 is the year the Sri Lanka government stopped controlling the domestic prices. The sample period of Philippines starts from 1970. The country abolished the

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[8] This plan is called Las Cuatro Estrategias (The Four Strategies).

[9] Before 1972, the ratio of the two rates was between 1.24(1971 I) and 2.19(1966 IV).
Source: Table 3.3, Liberalizing Foreign Trade vol.4.

Table 2-1. 2SLS Results

	α^s	β_1^s	β_2^s	β_3^s
Columbia:	-28.210^{**}	0.034	0.341	5.332^{**}
Sample period 1972-1992	(-2.386)	(1.478)	(0.761)	(2.043)
Sri Lanka:	-12513^{***}	0.067^{***}	2.020^{**}	0.921^{*}
Sample period 1978-1993	(-4.504)	(4.978)	(2.523)	(1.735)
Philippines:	-3.712^{**}	0.057^{**}	1787^{**}	2.914^{**}
Sample period 1970-1991	(-2.086)	(2.235)	(2.325)	(2.325)

Note: t-statistics are in parentheses.
* indicates 10% significance, ** indicates 5% significance, and *** indicates 1% significance.
D-W statistics are 1.41, 1.32 and 1.35 respectively.
The autocorrelation coefficient for Colombia is 0.298 with t-statistic of 1.961.

import controls and the exchange licensing in 1962, but the data for the share of imports in the country is only available from 1970. The sample period ends at 1991 due to the data unavailability for the import price.

Table 2-1 shows the result of a 2SLS regression of equation (2-28). The first row provides the results for Colombia. P_t^F is a weighted producer price index (PPI) of six major exporting countries to Colombia, four countries for Sri Lanka, and two countries for Philippines. Apparently, the Colombian data contains first-order serial correlation in the error term (Durbin-Watson statistic equals 0.913), so the Cochrane-Orcutt estimation method is employed.

The trend variable is often assumed to capture a large part of the increase in the amount of imports, but even with the trend in the regression, the coefficient of the ratchet variable is significantly positive in all regressions. The ratchet variable is much more effective on the imports for Colombia and Philippines than the price ratio itself. When the price in the Home Country goes up, many firms enter the Home market, but when the price goes down to the original level, not all of new firms withdraw. The significance of ratchet variable empirically supports the existence of asymmetric behavior of firms caused by the sunk costs.

One thing is clear from the estimations: the sunk cost of entry plays an important role in the foreign firm's decision making process. If some external shock, such as a reduction in tariff rate, increases the profitability, lots of foreign firms are attracted. When the shock is removed and the profitability goes back to its original level, the export supply will not go down to the level before the shock, but settles somewhere above it since some firms that entered while the shock was present will not exit from the market.

Concluding Remarks

In the first section, it is shown that the effect of a rise in the tariff rate on the domestic welfare is always negative when the number of varieties in the economy is allowed to change and the elasticity of demand depends on that number. Provided demand is not too elastic, the government tariff revenue increases with the tariff rate. The consumers always suffer, not only by higher prices of imports but also by reduced varieties. The loss is more than enough to offset the gain, and as a result, total welfare of the economy will always be reduced by an increase in the tariff. This analysis is based on the idea that any entrepreneur considering entering a foreign market will take account of the sunk cost of introducing a new good. The asymmetric responses of firms to a rise or a fall in the effective price, net of tariff and any other costs, creates hysteresis in their supply behavior. The significance of the sunk cost is justified empirically in the second section. The empirical evidence shows a notable hysteresis behavior in export supply, supporting the assumption made in the cost-benefit analysis in the previous section.

Finally, the monopolistic industry framework is simple to analyze, but may not capture the entire reality. It may be an interesting extension to introduce some game theoretic competition among suppliers.

CHAPTER 3
TRADE, WELFARE AND HYSTERESIS: THE CASE OF DEVELOPING COUNTRIES

Introduction

Hysteresis created by sunk costs is now a commonly applied concept to study supplier's behavior, mainly in investment and entry/exit decisions. Dixit (1989 and 1992) showed the significance of the existence of hysteresis caused by the entry cost for the investment activity when price (or the exchange rate in a model of an exporting firm) follows a Brownian motion causing uncertainty. Romer(1994) built a simple, but powerful model that shows the effect of a tariff imposed on imported inputs on the level of the country's outputs when there exists an entry cost. He showed that a tariff imposed on capital inputs will reduce the nation's output level by twice the tariff rate.

Many developing countries are not capable of producing manufactured goods, so they do not have domestically-made close substitutes to their imports. Thus, opening up the country to trade can mean introducing new products to consumers. Therefore, imported new goods may have a more significant relationship to the nation's welfare for the developing countries than for developed countries. Hence, the analysis in the following section targets a developing country which cannot domestically produce close substitutes to the imported goods. The analysis shows that hysteresis in the number of firms exporting to the country creates hysteresis in the nation's welfare level. When the government reduces the tariff rate, it may earn less tariff revenue and smaller consumer surplus from each imported variety, but as the total of all imported varieties increases, the magnitude of the nation's gain created by the increased varieties available is larger than the loss if the government raises the tariff rate by the same amount. This result implies, surprisingly, that if the government cuts its tariff rate once and then later raises it back to the original level, the nation enjoys a higher level of welfare afterwards than if the government had otherwise kept the tariff rate unchanged.

Cost-Benefit Analysis

Romer's model is, unfortunately, a one-period model, so hysteresis in the number of foreign firms entering the market is not present in the model itself. As an obvious result, he could not show what the tariff will do to the economy through the hysteresis effect of the number of goods available to the country. The following model shows the effect of a change in a tariff rate on the importing country's welfare when there is a significant level of fixed costs for firms exporting to the nation.

Let the inverse demand for an imported good *i* at time *t* be the function given by

$$P_{it}(x_{it}) = \alpha_i - \beta_i N_t x_{it}^2, \tag{3-1}$$

where α_i and β_i are positive constants, x_i is the quantity of good *i* produced at time *t*, and N_t is the total variety of goods imported at time *t*. Imported goods are not directly close substitutes or complements to each other. A change in demand for other goods does not affect the demand for good *i* directly but does affect it through a change in the total variety of goods imported: as N_t increases, the demand for any particular product decreases.

Exporting firms in foreign countries assume there will be no change in either the interest rate or the tariff rate in the future. Under some types of imperfect competition (in this case monopoly), a firm producing good i can be denoted as the firm i. Then the firm i's expected profit at time t=0 for infinite horizon, $E[\Pi_i]$, is

$$E[\Pi_i] = \frac{1}{r}\left[(1-\tau)P_{i0}(x_{i0})x_{i0} - c_i x_{i0}\right], \tag{3-2}$$

where r is the interest rate, τ is the uniform tariff rate at the initial time period, and c_i is a constant marginal cost for the firm i. For simplicity, assume symmetry across firms and demand for imports except fixed costs so that $\alpha_i = \alpha$, $\beta_i = \beta$, $x_{it} = x_t$, $c_i = c$ and $P_{it} = P_t$. Also, because firms do not anticipate any change in the tariff rate or the interest rate, they expect to produce the same amount in each time period, that is, $x_t = x$, and thus $P_t = P$ and $N_t = N$. Each firm takes the tariff rate and number of firms exporting to the country given, so they maximize the equation (3-2) with respect to the quantity they produce, x. Then the first order condition for this profit maximization problem is

$$\frac{1}{r}\left\{(1-\tau)\left[P(x) + P'(x)x - c\right]\right\} = 0. \tag{3-3}$$

Substituting the inverse demand function into the first order condition, the equilibrium quantity of the good each foreign firm exports is

$$x^*(\tau, N) = \sqrt{\frac{1}{3\beta N}\left(\alpha - \frac{c}{1-\tau}\right)}. \tag{3-4}$$

As the number of varieties imported increases (i.e. there are more foreign firms exporting to the country), the amount of each particular good imported decreases, but the total amount the country imports increases[10].

Each foreign firm faces a certain amount of fixed costs when it enters the market, such as the cost to set up a marketing network or advertising cost. Following Romer (1994), assume the fixed cost of entry is an increasing function of *i*. That is, the first firm that enters market (firm 1) has lower fixed cost than the 100th firm (firm 100). This is a very important assumption because, as will be demonstrated later, if the fixed cost of entry is same for every firm, all firms will enter or exit at the same time. The functional form of the entry cost is given by

$$FC^{e}(i) = \mu i^{5/2}, \tag{3-5}$$

where *e* stands for entry and μ is a positive constant. Moreover, firms are required to spend some fixed amount of maintenance costs in each time period in which they are in the market. This cost is also an increasing function of *i*, given by

$$FC^{m}(i) = \upsilon i^{5/2}, \tag{3-6}$$

where *m* stands for maintenance and υ is a positive constant which is smaller than μ. This maintenance cost can be interpreted as the cost of maintaining the marketing network or the cost of continuous advertisement. Therefore, the total fixed costs the firm i has to spend over time is

$$FC(i) = \left(\mu + \frac{\upsilon}{r}\right) i^{5/2}. \tag{3-7}$$

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[10] Total imports = $Nx^{*} = \sqrt{\frac{N}{3\beta}\left(\alpha - \frac{c}{1-\tau}\right)}$.

Thus, firm *i* will enter the market when it sees profitability after paying the fixed costs. That is, the entry condition for the firm *i* is

$$E[\Pi] = \frac{1}{r}\left[(1-\tau)P(x)x - cx\right] \geq FC(i) = \left(\mu + \frac{1}{r}\upsilon\right)i^{5/2}. \tag{3-8}$$

Here, it is obvious why the fixed costs must differ for each firm. If all firms face the same entry condition, a change in the profitability (perhaps through a change in the tariff rate) would not change the number of firms in the market unless it moves from below to above, or above to below the prohibitive level. Therefore, the market always contains either all the variety available in the world or nothing at all.

Because FC(*i*) is an increasing function of *i,* the last firm entering the market, say the *N*th, earns no economic profit after it pays the fixed costs. Hence, equation (3-8) becomes

$$\frac{1}{r}\left[(1-\tau)P(x)x - cx\right] = \left(\mu + \frac{1}{r}\upsilon\right)N^{5/2}. \tag{3-9}$$

Ignoring the integer constraint, the equilibrium number of varieties the country imports—which equals the number of foreign firms—at the initial time period can be calculated by using the first order condition for the profit maximization problem.

$$N^*(\tau)^{5/2} = \frac{1}{r\mu + \upsilon}\left[(1-\tau)P(x^*)x^* - (1-\tau)\left(P(x^*) + P'(x^*)x^*\right)x^*\right]. \tag{3-10}$$

Rearranging gives

$$N^*(\tau) = \left[\frac{1}{3\beta}\left(\alpha - \frac{c}{1-\tau}\right)\right]^{2/3}\left[\frac{2\beta(1-\tau)}{r\mu + \upsilon}\right]^{1/3}. \tag{3-11}$$

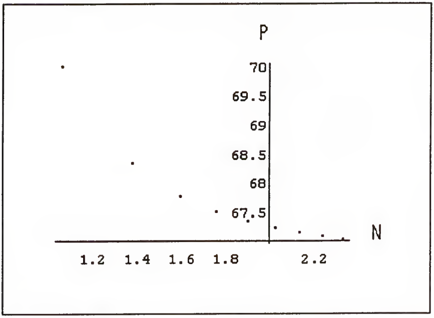

Figure 3-1. Relationship between N and P: Numerical Example

Figure 3-1 shows the relationship between the equilibrium number of firms (derived in equation [3-11]) and the consumer price (given by the equation [3-1]). As the tariff rate falls, the lower burden of the tariff attracts new firms to the market and the price falls.

Assume, now, the government of the importing country decides to change the tariff rate without any advance announcement. With this unanticipated change in the Tariff—and thus in the profit level—firms in the world are forced to decide whether to be in the market or outside of it. Firms that have already entered the market now face the choice between staying in the market or quitting, and firms which had not entered now consider whether to enter. Firms not yet in the market will consider entry when they see that entry is profitable. These firms have not previously entered because their fixed costs are too high to be compensated with the expected profit, $E[\Pi]$. The entry condition for a potential entrants is

$$E[\Pi] = \frac{1}{r}\left[\left(1-\tau^{h}\right)P(x^{h})x^{h} - cx^{h}\right] \geq \left(\mu + \frac{1}{r}\upsilon\right)i^{5/2}, \tag{3-12}$$

where *h* stands for the higher profit at a lower tariff rate and *i* must be larger than *N* at the initial period.

On the other hand, firms already in the market consider exiting from it when the expected profit is too low to cover their discounted maintenance costs. Their exit condition is

$$E[\Pi] = \frac{1}{r}\left[\left(1-\tau^{l}\right)P(x^{l})x^{l} - cx^{l}\right] \leq \frac{1}{r}\upsilon i^{5/2}\ , \tag{3-13}$$

where *l* stands for lower profit and *i* must be equal to or less than *N* at the initial period.

Notice the difference in the right hand side of two conditions, (3-12) and (3-13). This difference creates the hysteresis in the number of firms in the market. Also note that the entry condition after the change given by the equation (3-12) is very similar to the entry condition for the initial period. Thus, the equilibrium number of firms after an unanticipated rise in the profitability (due to a reduction in the tariff rate) is

$$N^{h}(\tau^{h}) = \left[\frac{1}{3\beta}\left(\alpha - \frac{c}{1-\tau^{h}}\right)\right]^{1/2}\left[\frac{2\beta\left(1-\tau^{h}\right)}{r\mu+\upsilon}\right]^{1/3}. \tag{3-14}$$

After an unanticipated fall in the profitability (due to a rise in the tariff rate), some firms already in the market exit from it and the new equilibrium number of firms is

$$N^{l}(\tau^{l}) = \left[\frac{1}{3\beta}\left(\alpha - \frac{c}{1-\tau^{l}}\right)\right]^{1/2}\left[\frac{2\beta\left(1-\tau^{l}\right)}{\upsilon}\right]^{1/3}. \tag{3-15}$$

To see the effect of asymmetric conditions given by equations (3-12) and (3-13), assume the government cuts the tariff by 20% of its original level in the period 1 and increases it by 25% in the following period[11]. That is, the tariff is reduced once and then brought back to the original level. Then the number of firms in each period can be represented as the following

$$N_0(\tau)=\left[\frac{1}{3\beta}\left(\alpha-\frac{c}{1-\tau}\right)\right]^{1/2}\left[\frac{2\beta(1-\tau)}{r\mu+\upsilon}\right]^{1/3}, \tag{3-16}$$

$$N_1(\tau)=\left[\frac{1}{3\beta}\left(\alpha-\frac{c}{1-0.8\tau}\right)\right]^{1/2}\left[\frac{2\beta(1-0.8\tau)}{r\mu+\upsilon}\right]^{1/3}, \text{ and} \tag{3-17}$$

$$N_2(\tau)=\left[\frac{1}{3\beta}\left(\alpha-\frac{c}{1-\tau}\right)\right]^{1/2}\left[\frac{2\beta(1-\tau)}{\upsilon}\right]^{1/3}, \tag{3-18}$$

where N_0 is the initial number of firms, N_1 the number while the tariff is low, and N_2 the number after the tariff has been restored to its initial level.

If there is no effect of asymmetry, the equilibrium number of firms in the original period and the number in period 3 must be the same because the tariff rate is the same. However, as it is obvious from the discussion above, the equilibrium number of firms increases from N_0 to N_2 [12]. When the government reduces a tariff, many firms which have higher fixed costs enter the market, but when the government increases the tariff, not all of the new comers exit because they have already paid the entry costs.

Now, move to a cost-benefit analysis of a change in the tariff rate. By importing products from the outside, consumers enjoy consumer surplus created from the goods and the government collects tariffs from the foreign producers, which go to the domestic economy. Therefore, the welfare created by imports consists of two parts: the consumer surplus and the tariffs. The consumer surplus from an imported good is expressed as the following

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[11] Because this is an infinite time period model, when the tariff is reduced, when it is set back to the original level and how long it is between the changes do not create a significant impact on the qualitative results.

$$CS_t = \int_0^{x_t^*} P(x)dx - P(x_t^*)x_t^*$$

$$= \frac{2}{3}\beta N_t^* x_t^{*3}. \tag{3-19}$$

The tariff revenue from an imported good is

$$TR_t = \tau_t P(x_t^*)x_t^*$$

$$= \tau_t\left(\alpha - \beta N_t^* x_t^{*2}\right)x_t^*. \tag{3-20}$$

These are the welfare created by a single good, but the economy has N_t imported goods available in the market. Thus, the total welfare this country enjoys by importing is

$$W_t^I(\tau_t) = N_t^*(\tau_t)(CS_t + TR_t). \tag{3-21}$$

The total welfare multiplies the welfare created by each good by the number of available goods because of the symmetry in the demand and supply of imported goods, except for the fixed costs.

Following the earlier case, assume the government reduces the tariff by 20% in the first period and then increases it back to its original level in the following period. The total welfare in those periods are

$$W_0^I = \frac{2}{27}\sqrt{\frac{3}{\beta}}\left(\alpha - \frac{c}{1-\tau}\right)^{7/4}\left[\frac{2\beta(1-\tau)}{r\mu+\upsilon}\right]^{1/6}$$

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[12] $N_2 - N_0 = \left[\frac{1}{3\beta}\left(\alpha - \frac{c}{1-\tau}\right)\right]^{1/2}\left[2\beta(1-\tau)\right]^{1/3}\left(\frac{1}{\upsilon} - \frac{1}{r\mu+\upsilon}\right)^{1/3} > 0.$

$$+\frac{\tau}{9}\sqrt{\frac{3}{\beta}}\left(\alpha-\frac{c}{1-\tau}\right)^{3/4}\left[\frac{2\beta(1-\tau)}{r\mu+\upsilon}\right]^{1/6}\left[2\alpha+\frac{c}{1-\tau}\right], \tag{3-22}$$

$$W_1^I=\frac{2}{27}\sqrt{\frac{3}{\beta}}\left(\alpha-\frac{c}{1-0.8\tau}\right)^{7/4}\left[\frac{2\beta(1-0.8\tau)}{r\mu+\upsilon}\right]^{1/6}$$

$$+\frac{0.8\tau}{9}\sqrt{\frac{3}{\beta}}\left(\alpha-\frac{c}{1-0.8\tau}\right)^{3/4}\left[\frac{2\beta(1-0.8\tau)}{r\mu+\upsilon}\right]^{1/6}\left[2\alpha+\frac{c}{1-0.8\tau}\right], \text{ and} \tag{3-23}$$

$$W_2^I=\frac{2}{27}\sqrt{\frac{3}{\beta}}\left(\alpha-\frac{c}{1-\tau}\right)^{7/4}\left[\frac{2\beta(1-\tau)}{\upsilon}\right]^{1/6}$$

$$+\frac{\tau}{9}\sqrt{\frac{3}{\beta}}\left(\alpha-\frac{c}{1-\tau}\right)^{3/4}\left[\frac{2\beta(1-\tau)}{\upsilon}\right]^{1/6}\left[2\alpha+\frac{c}{1-\tau}\right]. \tag{3-24}$$

The first line in each equation is the consumer surplus and the second line is the tariff revenue from all imported products.

As in the case for the equilibrium number of firms in the market, the total welfare in the nation after the tariff rate is back to the original level is larger than the initial level of the total welfare. This result is illustrated in the figure 3-2. The reduced tariff in the period 1 attracts many firms into the market, and after the tariff rate is raised back to its initial level, some of new firms stay. As a result, even after the tariff returns to the beginning rate, consumers enjoy more varieties of goods. Although this increased number of firms decreases the demand for each good, so that the new equilibrium quantity of each good imported is lower than the original level (which creates less consumer surplus and tariff revenue[13]), the total volume of imports increases as it has been shown earlier. The reason for this increase is clear; for intuition, consider the horizontal distance

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[13] See Appendix A for more details.

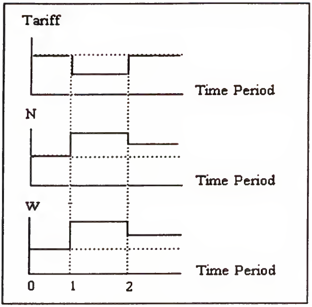

Figure3-2. Hysteresis in τ and N

model. As the number of sellers on a straight main street increases, each customer located on the street faces less transportation costs to get to the seller and so has more purchasing power. It is the same story in the model at hand. As more varieties are imported, each consumer has to "travel" less distance on his/her preferences to a good because these goods are not perfect substitutes for each other.

When the government cuts the tariff rate, it loses some tariff revenue from each good but the total tariff revenue is larger because there exists more sources to collect the tariff. Similarly, consumers enjoy more varieties and create larger consumer surplus. The

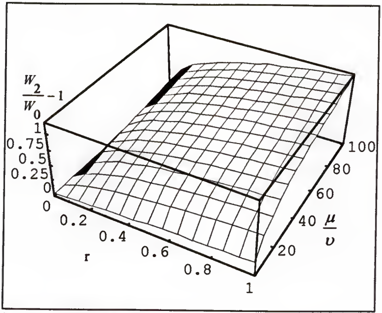

Figure 3-3. Welfare Gain

magnitude of the gain by lowering the tariff is larger than the magnitude of the loss from raising the tariff, which creates the hysteresis in the welfare level.

Figure 3-3 shows the proportion of the gain in period 2 by cutting the tariff rate once and increasing it to the initial level compared to the original welfare level. The gain totally depends on the interest rate, with which firms and consumers calculate the discounted values of profit and welfare, and the ratio of the fixed costs of entry and the fixed maintenance costs foreign firms required to spend[14]. If the ratio is 100 and the interest rate is 10 percent, the welfare after the tariff rate is settled to the initial level is almost 50 percent higher than the original welfare level. It can be strange at the first sight that the gain does not depend on the proportion of the tariff reduction at all. It is, however, actually reasonable because the equilibrium number of the foreign firms in the market, and the quantity of their exports, do not depend on the tariff rate in the previous period. If the model was not infinite-horizon and thus the length of the period the tariff rate was reduced mattered, then the tariff rate in the all periods would affect the welfare change.

Concluding Remarks

The previous section showed how a nation can benefit by reducing the tariff rate: the lower tariff attracts many firms which would not enter the market in the earlier period because of their high fixed costs. These firms introduce new consumer products to the economy. Then, even when the government raises the tariff back to its original level, some of these new firms stay in the market and continue to provide an extra source of consumer surplus and tariff revenue.

Although the assumptions of unanticipated tariff rate changes combined with otherwise perfect foresight gives simple and intuitive result, it restricts the situation. It may be an interesting extension of this analysis to introduce imperfect or asymmetric information into this framework. Then the model can be expanded to analyze the government's decision of when to change the tariff rate.

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[14] $\frac{W_2}{W_0} - 1 = (1 + r\sigma)^{1/6} - 1$ where $\sigma = \frac{\mu}{\upsilon}$.

CHAPTER 4
CONSUMER UTILITY, VARIETY AND APARTMENT RENTS

Introduction

Estimating the implicit prices of attributes of both single-family and multi-family housings has been a major interest of business and academic researches. Estimated prices can be used to evaluate an existing property, to design new properties to best fit the market need, and analyze the trends of price changes. Shelter expenditure is one of major components of GDP, so the estimated prices become an important input to calculate the price level across time and geographic places. For those purposes, accuracy in price functions and price indexes have been intensively sought after.

The conventional and widely accepted monocentric urban economic model posits that the distance to the city center from the property is an important price-determining factor. Essentially, this approach assumes that the city center is also the employment center, so housing is built farther away from the center so that the transportation cost to the work place increases, the price of the shelter must decrease. This model contains a well-developed theory and offers many useful insights, but there are several shortcomes. First, the city center is not the only place which offers employment opportunities. Many studies have demonstrated the significance of urban subcenters (See, for example, Waddel, Berry and Hoch [1993]). However, modeling and estimating multinodal urban structure turns out to be extremely complicated. Second, the majority of trips made by households are not for the work but for other daily activities such as grocery shopping and entertainment, which are usually done within the immediate neighborhood.

While the urban center model is widely accepted, empirical tests of it are scarce. This chapter, therefore, helps fill two gaps. First, it tests the conventional model using data collected from over 3,000 observations in Florida, a hedonic regression with standard structural and locational characteristics for apartments is estimated[15]. This regression clearly shows that apartment rents start increasing from six years after construction and continue to be above the initial level for about one decade. Why in a hedonic regression, controlling for many apartment unit and complex amenities, is an apartment built in 1985 rented at a higher rent than an apartment built in 1993? The standard urban economic theory points out that the answer may be because the city in which the apartment is located expands outward and so the value of the location of the apartment keeps rising as it becomes relatively closer to the city center compared to the border of the city. Hence, this result qualitatively supports the standard urban-center theory. However, quantitatively the standard urban economic theory is not enough to explain the dramatic rent hike. A twelve-year old apartment is far more expensive than the standard theory predicts it would be. To overcome this problem of the standard monocentric urban model, this chapter fills a second gap in the theory by adding the concept of "activity circles," with product variety, to the standard urban model. The concept of an "activity circle" states that a resident values the varieties of activities available in the immediate neighborhood of the apartment and so the extent of these activities affects the apartment rent. The empirical results suggest that this concept does influence rents.

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[15] Apartment rents provide a better test than do housing prices because they pertain to a given year value rather than being the discounted value of a future flow of services.

The rest of this chapter is constructed as follows: section two explains important factors in the shelter market and describes a widely accepted estimation technique introduced by Rosen (1974). Section three gives the description of the regression model used to estimate the standard urban economic model and discusses the result. Section four introduces and empirically justifies the concept of "activity circle." The last section concludes the study and summarizes some of the key implications of the results.

Heterogeneity of Housing Services and Hedonic Price Functions

The difficulty of modeling, or estimating, the value of each characteristic of a housing market lies in the fact that the specific attributes never appear explicitly in market transactions. A housing price or an apartment rental rate includes the value of all the flows of services received by a resident, such as the size of the rooms, the availability of a washer/drier or a fitness center, and the accessibility to any activity centers, as well as the capital value of the house in the case of an owner-occupied house. Furthermore, housing and apartment units are known for their extreme heterogeneity, a characteristic that makes ordinary estimation procedures very difficult. No housing or apartment units are exactly alike. The traditional saying of "a potato is a potato" does not apply to the housing or apartment markets. A three-bedroom apartment with 1,500 square feet in New York is not the same as a three-bedroom apartment of the same size in Key West, FL. Even if all of the available amenities are the same, the value for residents for those two apartments is different simply because they are located in different places.

Rosen (1974) suggested that a hedonic regression approach should be utilized to overcome this problem. Rosen's technique has come to be widely accepted. A hedonic model estimates the implicit marginal value of each service by creating a "base" model of apartment with a (relatively) small number of observable attributes. Then it is possible to determine the marginal values for each specific attribute.

It is important to realize that a hedonic price function is an envelope function comprised of both demand functions by households and supply functions by landlords and developers. Therefore, the resulting marginal value of an attribute measures neither the willingness to pay by potential residents nor the minimum price suppliers need to receive to supply the attribute[16]. Instead, the estimated marginal values are the market prevailing equilibrium values under whatever level of competition prevails in the market.

Hedonic Estimation Results

In order to determine if there is a rent hike in the early period of the life of apartment buildings, a standard hedonic regression is performed. This regression uses rent data collected by the Bureau of Economic and Business Research at the University of Florida for the Florida Price of Living Index program. The data have been collected from over 3,500 apartment complexes in Florida[17]. The great advantage of local data is the large number of observations, especially in comparison to the typical number of 100 or so used in other studies. Also, Florida has a wide range of population growth rates, ranging from three or four counties that were among the ten most rapidly growing areas in the United States (such as Collier County, Lee County, St. Louis County and Charlotte

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[16] For a more detailed discussion, see Rosen (1974) and Rothenberg et al., Chapter 3 (1991).

[17] For more detailed explanation of the data set, see Appendix B.

County) to mature urban areas (such as Dade County) to slow-growing areas (like some counties in North Florida). The data also have some large but still rapidly growing areas (such as Orange County).

Weddell, Berry and Hoch (1993) point out the potential problem of high collinearity between the age of building and distance from the city center, and suggest using sets of dummy variables for the distance variables to minimize the problem of correlation. Therefore, the hedonic rent function estimated in this study takes the following form:

$$R_i(X,Y,A,C,D;\alpha) = \alpha_0 + \sum_{j=1}^{n} \alpha_{1j} X_{ji} + \sum_{k=1}^{m} \alpha_{2k} Y_{ki} + \sum_{l=1}^{p} \alpha_{3l} A_{li} + \sum_{r=1}^{q} \alpha_{4r} C_{ri} + \sum_{s=1}^{v} \sum_{u=1}^{z} \alpha_{su} D_{sui} + \varepsilon_i \,, \qquad (4\text{-}1)$$

where R_i is a natural log of the rent for apartment *i*, *X* is a vector of unit characteristics, such as the number of bedrooms, *Y* is a vector of complex characteristics, such as the availability of a swimming pool, *A* is a set of age dummy variables, *C* is a set of county dummy variables and *D* is a set of distance dummy variables. The county dummy variables help serve as a proxy for multi-nodal central business district (CBD) within the State.

The unit characteristic variables are structural characteristics, or amenities, that are unique for each unit of apartment. For the unit characteristics, the "*X*" variables, the following variables are included in the regression equation.

FURN : takes 1 if the apartment is furnished, 0 otherwise
CABLE : takes 1 if the apartment is provided with cable TV, 0 otherwise
DISH : takes 1 if the apartment has a dishwasher, 0 otherwise
STORAGE : takes 1 if the apartment has extra storage space, 0 otherwise
FAN : takes 1 if the apartment has a ceiling fan, 0 otherwise

MWAVE : takes 1 if the apartment has a microwave, 0 otherwise
WD : takes 1 if the apartment has a washer/drier, 0 otherwise
PATIO : takes 1 if the apartment has a patio, 0 otherwise
BEDR : number of bedrooms in the apartment
BATH : number of bathrooms in the apartment
LSIZE : natural log of the size of the apartment, in square foot

The complex characteristics are structural characteristics or amenities that are shared by all residents in the same complex. For complex characteristics, the "*Y*" variables, the following variables are included in the estimation model.

GATE : takes 1 if the apartment has a secured front gate, 0 otherwise
COVERD : takes 1 if the apartment has covered parking lots, 0 otherwise
GARAGE : takes 1 if the apartment has garage parking, 0 otherwise
OSMNG : takes 1 if the apartment is provided some on-site management, 0 otherwise
EMG24 : takes 1 if the apartment is provided a 24-hour emergency service, 0 otherwise
POOL : takes 1 if the apartment has a swimming pool, 0 otherwise
FITCTR : takes 1 if the apartment has a fitness center, 0 otherwise
CLBHOUSE : takes 1 if the apartment has a clubhouse, 0 otherwise
TENNIS : takes 1 if the apartment has a tennis court, 0 otherwise
PET : takes 1 if the resident is allowed to keep a pet, 0 otherwise
LUNITS : natural log of the number of units in the complex
MOTEL : dummy variable for a representative motel chain

The last variable, MOTEL, requires some explanation. During the rent survey, the manager of each apartment complex is asked to choose a type of hotel/motel that best represents the complex[18]. The choices are: i) Motel 6, ii) Scottish Inn, iii) Days Inn, iv) Holiday Inn, v) Best Western, vi) Hilton, vii) Marriot and viii) Omni. An average overnight rate index for each chain is calculated using the Holiday Inn as the base. The base is indexed as zero, so chains with lower rate than the Holiday Inn have negative vales for the index. Then, the calculated index is used as the variable MOTEL: i.e. if a manager chooses the Holiday Inn as a representative motel, then the apartment is assigned a value zero (since the motel is the base) for the variable MOTEL. This variable is meant to proxy any omitted or unobservable variables. The advantage of this method over other rating systems, for example, rating the complex according to 1-10 scale, is that this way is less dependent on varying individual standards. Having a standard guideline to compare helps the respondents to be less subjective. The hotels and motels listed here are well known and have varying combinations of price and quality[19].

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[18] For the importance of subjective measure variables, see Lang and Jones (1979).

The age of apartments, the "*A*" variable, is divided into following 17 categories. The variable takes on the value of one if the apartment's age fits that category and zero otherwise.

AGE2 : apartments built in 1996 or later
AGE4 : apartments built between 1994 and 1996 (latter exclusive)
AGE6 : apartments built between 1992 and 1994 (latter exclusive)
AGE8 : apartments built between 1990 and 1994 (latter exclusive)
AGE10: apartments built between 1988 and 1990 (latter exclusive)
AGE12: apartments built between 1986 and 1988 (latter exclusive)
AGE14: apartments built between 1984 and 1986 (latter exclusive)
AGE16: apartments built between 1982 and 1984 (latter exclusive)
AGE18: apartments built between 1980 and 1982 (latter exclusive)
AGE20: apartments built between 1978 and 1980 (latter exclusive)
AGE22: apartments built between 1976 and 1978 (latter exclusive)
AGE24: apartments built between 1974 and 1976 (latter exclusive)
AGE26: apartments built between 1972 and 1974 (latter exclusive)
AGE30: apartments built between 1968 and 1972 (latter exclusive)
AGE40: apartments built between 1958 and 1968 (latter exclusive)
AGE50: apartments built between 1948 and 1958 (latter exclusive)
AGEOLD: apartment built prior to 1948

The distance or locational variables, the "*D*" variables, are the distances to the apartment complex from the nearest city center with population of 50,000 or more; from the nearest coast; from the nearest golf course; and from the nearest k-12 (elementary, middle and high) school. In addition, a dummy variable UPSCALE is included as a part

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[19] See Denslow and Scoggins (1997) for the relatiojnship between the amenities of apartments and the result of motel comparison.

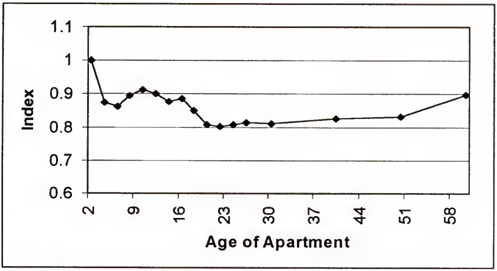

Figure 4-1. Aging Effect on Apartment Rent

of locational characteristics. This variable takes the value of one if the apartment is located in upscale neighborhood. The specific distance dummies used in the model are as follows.

CITY1 : takes 1 if the apartment is located at less than or equal to1 mile away from the city center, 0 otherwise
CITY2 : takes 1 if $1 <$ distance from the city center ≤ 2
CITY4 : takes 1 if $2 <$ distance from the city center ≤ 4
CITY7 : takes 1 if $4 <$ distance from the city center ≤ 7
CITY10: takes 1 if $7 <$ distance from the city center ≤ 10
CITY15: takes 1 if $10 <$ distance from the city center ≤ 15
CITY20: takes 1 if $15 <$ distance from the city center ≤ 20
CITY30: takes 1 if $20 <$ distance from the city center ≤ 30
CITY50: takes 1 if $30 <$ distance from the city center ≤ 50
CITYFAR: takes 1 if $50 <$ distance from the city center
GOLF0: takes 1 if the apartment is located at less than or equal to 0.5 mile away from a golf course , 0 otherwise
GOLF1: takes 1 if $0.5 <$ distance from a golf course ≤ 1
GOLF2: takes 1 if $1 <$ distance from a golf course ≤ 2
GOLF3: takes 1 if $2 <$ distance from a golf course ≤ 3
GOLF5: takes 1 if $3 <$ distance from a golf course ≤ 5
GOLFFAR: takes 1 if $5 <$ distance from a golf course
COA0 : takes 1 if the apartment is located at less than or equal to 0.5 mile away from a coast, 0 otherwise
COA1 : takes 1 if $0.5 <$ distance from a coast ≤ 1

COA2 : takes 1 if 1 < distance from a coast ≤ 2
COA3 : takes 1 if 2 < distance from a coast ≤ 3
COA5 : takes 1 if 3 < distance from a coast ≤ 5
COA7 : takes 1 if 5 < distance from a coast ≤ 7
COA10: takes 1 if 7 < distance from a coast ≤ 10
COA15: takes 1 if 10 < distance from a coast ≤ 15
COA20: takes 1 if 15 < distance from a coast ≤ 20
COA30: takes 1 if 20 < distance from a coast ≤ 30
COAFAR: takes 1 if 30 < distance from a coast
SCH0 : takes 1 if the apartment is located at less than or equal to 0.5 mile away from a school, 0 otherwise
SCH1 : takes 1 if 0.5 < distance from a school ≤ 1
SCH15: takes 1 if 1 < distance from a school ≤ 1.5
SCH2 : takes 1 if 1.5 < distance from a school ≤ 2
SCH3 : takes 1 if 2 < distance from a school ≤ 3
SCH4 : takes 1 if 3 < distance from a school ≤ 4
SCHFAR: takes 1 if 4 < distance from a school

Finally, there are 67 counties in Florida, of which two counties apparently have no licensed apartment complexes. As mentioned, the county dummies are included, in part, to help serve as a proxy for counties with multiple large cities. In addition, of course, they capture any county specific factors, such as differing price levels.

Figure 4-1 shows the estimation result on the age coefficients in equation (4-1)[20]. It is clear that there is an increase in the rent after a few years. The very high rent for the newest apartments might be explained by its strong competitiveness of being "brand new" against close substitutes, that is, older apartments.

The Problem of the Standard Urban Economic Model

The standard urban economic model proposed by Capozza and Helsley (1989 and 1990) offers a possible explanation for the seemingly puzzling rent hike between six and twelve years after construction[21]. The model (hereafter referred as the C-H model) argues that the rent for otherwise equal units[22] is determined by the distance to the city center compared to the distance between the city border and the city center. The relative distance shows the savings in commuting cost, by living closer to the city center, which is assumed to be the place of employment. A resident is willing to pay some extra rent to live closer to the city center as long as the additional amount does not exceed the savings in the commuting cost.

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[20] A complete table of the results for equation (3-1) is listed in table B-1 in Appendix B.

[21] DiPasquale and Wheaton (1997) introduce the Cappoza-Helsley model in a very straight forward way in Chapter 3.

Assume all residents in the city earn annual income of y, and spend all of it in the rent (R), commuting costs (c) and other goods and services (x). Let the average price of other goods and services be the numeraire. Then the budget constraint for a household is

$$y = x + R + c\,. \tag{4-2}$$

Following Cappoza and Helsley, the annual commuting cost for a resident living d miles away from the city center is $c = kd$, where k is the annual commuting cost per mile. Then the budget constraint in equation (4-2) becomes

$$y = x + R + kd\,. \tag{4-3}$$

Because the annual income is assumed to be equal for all households in the city, the budget constraint for a household living d miles away from the city center is the same as the one for a household living at the city's outside border. If the radius of the city is b miles, then it is true that

$$y = x_d + R_d + kd = x_b + R_b + kb\,, \tag{4-4}$$

where the subscript d denotes a household d miles away from the city center and the subscript b indicates a household on the city's border.

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[22] The C-H model assumes all apartments in the city have the same characteristics except the distance from the city center.

Consumers gain utility from the quantity of other goods and services consumed (x) and the quantity of amenities in the apartment (q). Because the amenities are assumed to be the same for all apartments in the city, utility depends only on the value of x. In order to keep the utility for all residents in the city equal (otherwise people will move to the location which yields the highest utility), it must be the case that

$$U(x_d) = U(x_b)\,. \tag{4-5}$$

This implies that the quantity of other goods and services consumed is the same for all households in the city. Therefore, from the budget constraint in equation (4-4)

$$R_b + kb = R_d + kd\,. \tag{4-6}$$

Thus, the difference in the rents is

$$R_b - R_d = k(b-d)\,. \tag{4-7}$$

That is, the extra rent the resident has to pay to live closer to the city center equals the savings in the commuting cost. By definition, the rent at the city's border must be the sum of the agricultural rent plus the cost of construction, so R_b is constant as the city grows outward. Thus, the over-time movement of the rent at location d can be expressed by differentiating equation (4-7) with respect to time, t:

$$\frac{d(R_d - R_b)}{dt} = \frac{d(R_d)}{dt} = k\frac{db}{dt}\,. \tag{4-8}$$

That is, as the city grows and the border expands outward, the savings in the commuting cost becomes larger moving the rent at any location inside the border higher. Thus, C-H model predicts that the rate of rent increase over time depends on the rate of the city's

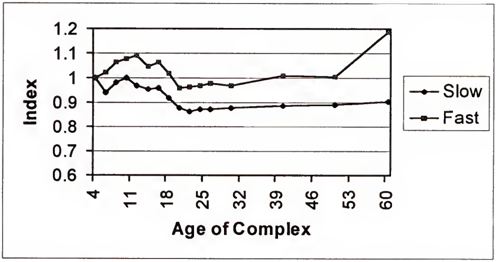

Figure 4-2. Aging Effects for Slow and Fast Growing Counties

expansion. The more rapidly the city expands, the faster the rent grows, as equation (4-8) implies[23].

Surprisingly, although this model has been accepted widely and used in many urban economics textbook, little empirical research has been published to confirm its validity. The relationship between the rate of city expansion and the rate of rent increase implies that the more rapidly a city grows, the more rapidly the rent increases. In order to perform a test of this hypothesis, the data set was divided into two groups: fast growing counties and slower growing counties.

Figure 4-2 shows the results for the age coefficients with two sets of age dummy variables: one for the faster growing counties and one for the slower growing counties. Those two sets of age dummies are included in one regression equation to keep the marginal values of other amenities equal. Age dummy variables for apartments two-year

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[23] The mathematical relationship proposed by the C-H model is $\%\Delta R \equiv \frac{\partial R}{R} = \frac{kb}{R}\frac{\partial b}{b} \equiv \frac{kb}{R}\%\Delta b$ where Δ is "the change in" the following variable. This result can be derived by rearranging equation (4-8).

Table 4-1. Comparison of Estimated Rents

Age	Hedonic Index	C-H Model	Difference
4	488.81	488.81	N/A
6	500.29	493.59	6.70
8	520.12	498.37	21.75
10	527.66	503.16	24.50
12	533.41	507.94	25.47
14	511.69	512.73	-1.04

old or newer are dropped from the regression because other factors, such as advantages of being brand new, are included. The estimated age index variables for the faster growing counties lie above the index variables for the slower growing counties for all ages (except for AGE4 dummy, which is the base). The significance of the difference between the two groups is confirmed by following F-test[24].

$$F = \frac{\left(R^2 - R_*^2\right)/j}{\left(1 - R^2\right)/(n-l)} = 2.8378, \tag{4-9}$$

where R_*^2 is the R^2 value for the restricted model estimation (the restriction is that the age index variables for faster and slower growing counties have the same coefficients), j is the number of restrictions and n-l is the degrees of freedom. The critical value for $F_{(15, 3275)}$ is 1.67, so this test rejects the equality of the coefficients. Hence, the regression estimates support the C-H model insofar as there is a significant difference in the rent index for the faster growing and slow growing counties.

Although the C-H model is qualitatively supported, the estimated rent hike is higher than the model predicts. Table 4-1 presents the estimated rent from the hedonic model, equation (4-1), and C-H model for the years during which the rent is estimates to be above the initial level. The first column in table 4-1 is the age group, the second column shows the rent estimated from equation (4-1) and the third column shows the rent calculatd using the C- H model. The annual commuting cost per mile, *k*, is taken from DiPasquale and Wheaton (1997) as 200; the rate of city border expansion, $\partial b/b$, is calculated from the rate of population growth between 1980 and 1990 listed in the Florida Statistical Abstract (1995)[25]; and, the distance between the city center and the city border is estimated from the data by averaging the furthest apartment from the city center in each county. The resulting numbers for the C-H calculation are: $k = 200$, $\partial b/b \equiv \dot{b} = 0.01435$, and $b = 14.1088$.

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[24] See Greene (1993) Chapter 7 for the details of this F-test.

In table 4-1, the first entries have the same value for both hedonic index and C-H model. The C-H model needs a starting rent value and so the estimated rent from the hedonic model is used for that value. The difference between those two sets of estimated rent, listed in the last column, indicates that although the C-H model is qualitatively correct, quantitatively it underestimates the rapid rent increase. There must be something additional to the factors emphasized by the C-H model that causes the rent to increase so rapidly in the early stage of an apartment building's life. The next section discusses such an extension.

Activity Circle, Consumer Utility and Rent

The concept I introduce is not entirely novel, but it has never been formally addressed or combined into a model dealing with rent changes. Assume there is an apartment at the edge of a city at time t_0. The distance to the city center from this apartment is b_0. Therefore, the annual cost of commuting to the city center for a resident

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[25] The estimation uses the result that $\frac{\partial b}{b} = \frac{1}{2}\frac{\partial POP}{POP}$.

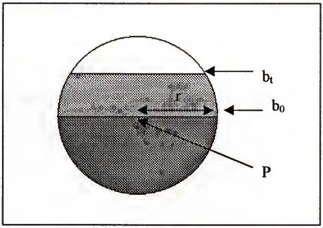

Figure 4-3. Activity Circle

of this apartment is kb_0. The C-H model assumes that this cost is the only commuting cost that affects the rent. However, the city center is not the only place a resident's daily activity takes place. He/she needs to go grocery shopping, perhaps to eat out, to see a movie, or go to a sports bar. For these activities, most people do not go to the city center, but go to somewhere in their neighborhood. Therefore, the availability, or the variety, of those daily activities in the local neighborhood affects the resident's utility. As the previous two chapters show, the consumers' utility depends not only on the quantity of goods and services consumed, but also on the number of varieties available. Put simply, it is preferable to have locally five different restaurants to choose from than to have only one. Indeed, this point is the reason that some apartment complexes advertise their nearness to shopping malls or community parks.

Assume the city density is uniform, so there are the same number of varieties of activities available in one acre of land anywhere within the city. Thus the number of activities can be measured by the size of the neighborhood city area. The utility function now becomes

$$U(x, v(A)) = u(x, A), \tag{4-10}$$

where A is the size of the neighborhood area and $v(A)$ is the function showing the number of activity varieties available for the resident. The budget constraint stays the same.

Figure 4-3 illustrates the concept of an "activity circle." The apartment is located at point P, and, by assumption, the resident's daily activity takes place only within r miles from the apartment. That is, the resident only obtains utility from activities within a circle of radius r around his or her apartment. The circle in figure 4-3 has a radius of r, so it represents the resident's activity circle, in which his/her daily activities take place. At time t_0, the apartment is located at the city border, so daily activities are available only in the existing half of the activity circle (the dark shaded area in figure 4-3). As the city grows, the border is pushed outward (upward in the diagram). The area within the resident's activity circle becomes larger and larger (the light shaded area plus dark shaded area in figure 4-3), until the city's border reaches the far end of the circle. After that time, the growth of the city does not affect the area of the activity circle because it is already as large as possible.

For a resident at location d, within the city, the activity circle is larger than the one for a resident at the city's border. Then, to be in equilibrium, the following condition must be met at all the time.

$$u(x_d, A_d) = u(x_b, A_b). \tag{4-11}$$

Because $A_d \neq A_b$ (assuming $d \neq b$), if the rents are determined by the simple C-H model, the utility gained from locating within the city so as to enjoy a full activity circle is higher than that gained from locating on the city edge, with only one half of an activity circle. With this in mind, the C-H rent function that only considers the distance to the city center and the savings in the commuting cost to the center does not yield an equilibrium with equalized utility throughout the city. To be in equilibrium, each resident has to receive the same level of utility, so the rent must absorb the extra utility created by not being at the border. Hence, it is no longer true that all residents consume the same amount of other goods, x. Essentially, more variety compensates for less quantity. With a lower level of consumption in the inner city compared to at the border, required for equilibrium, the resident within the city has to pay a higher rent. (Notice that the number of varieties does not enter into the budget constraint.)

To see the change in the rent over time caused by the change in the activity circle, first rearrange the budget constraint to get the rent function.

$$R_d = (y - kd) - x_d. \tag{4-12}$$

Differentiate this with respect to t to yield

$$\frac{dR_d}{dt} = -\frac{dx_d}{dt}. \tag{4-13}$$

The change in x_d over time can be found by totally differentiating the utility function in equation (4-10).

$$\frac{dx_d}{dt} = \frac{u_A}{u_x}\frac{dA_d}{dt}, \tag{4-14}$$

where $u_i = \dfrac{\partial u(x_d, A_d)}{\partial i}$ for $i = x_d$ and A_d. If we assume that the utility function is separable and the ratio of the two marginal utilities is constant, then equation (4-12) shows the time path of the rent at location d depends solely on the time path of the developed area in the activity circle.

Table 4-2. Activity Circle Augmented Comparison of Estimated Rents

Age	Hedonic Index	C-H Model	Difference	A_t-A_4
4	488.81	488.81	0.00	0.00
6	500.29	493.59	6.70	2.43
8	520.12	498.37	21.75	4.74
10	527.66	503.16	24.50	6.83
12	533.41	507.94	25.47	8.515
14	511.69	512.73	-1.04	9.236

The developed area within the activity circle at time *t*, A_t, equals[26],

$$A_t = \frac{r^2\pi}{2} + r^2 \sin^{-1}\left(\frac{b_0\left((\dot{b})^t - 1\right)}{r}\right) + b_0\left((\dot{b})^t - 1\right)\left[r^2 - b_0^2\left((\dot{b})^t - 1\right)^2\right]^{\frac{1}{2}}. \tag{4-15}$$

Because the first derivative of the above function is positive and the second derivative is negative (see Appendix B for the detail), A_t is a monotonically increasing function with a concave shape. Therefore, the change in the rent over time caused by the change in the number of varieties available is also a monotonically increasing function with a concave shape. Notice that this is only the effect from the activity circle. The entire time path of the rent is the sum of the effect from the activity circle and the effect from the saving in the commuting cost, which is expressed in equation (4-7).

For simplicity, assume the dollar amount of utility created by the activity within the activity circle is a linear function of the area. Table 4-2 is the same as table 4-1 with the addition of the fifth column, the additional area within the activity circle compared to the area at the fourth year. The activity circle is assumed to be half full at year zero. The

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[26] For calculation purpose, the city border within an activity circle is assumed to be a straight line although a city has a circular shape. It can be interpret that an activity circle is so small compared to a city, so the fragment of the border which lies in it is almost a straight line.

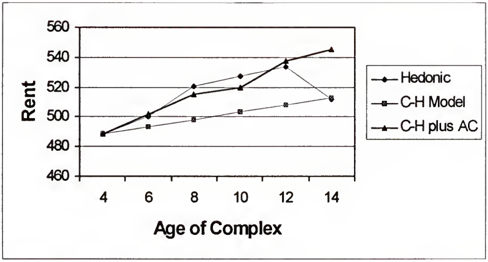

Figure 4-4. Rent Comparison

same values are used for the distance from the city center to the border at the initial time period, b_0 and the rate of city expansion, $\dot{b}$. The radius of a resident's activity circle, *r*, is assumed to be 3 miles. If the additional activity circle area is multiplied by a number between 3 and 4, the value is very close to that of the difference between the estimated rent from the hedonic index and from the C-H model. Notice that the far end of the activity circle is reached by the city border after thirteen and a half years, so that after that point in time, there is no utility increase associated with a rise in the number of available activity variety.

Figure 4-4 shows a comparison between the hedonic rent, the C-H model rent and the C-H model rent plus the activity circle. The multiplier used to calculate the dollar amount of utility gain from the activity circle is 3.5. The estimated rent that includes the activity circle effect is closer to the rent estimated from hedonic regression. From this result, it is clear that the C-H model augmented with the concept of an activity circle can better explain the rent hike in the first one and a half decades after the complex is constructed.

Recall that both the C-H model and the activity circle model do not consider the effect of depreciation of the apartment building. The effect is small at the beginning but must gets larger as time goes by. This effect may explain the difference between the theoretical models and the estimation in the last few years. Lane, Randolph and Berenson (1988) estimated the rate of depreciation for major metropolitan areas for CPI adjustment purpose[27]. Their results range from 0.16% for fast growing South cities to 0.36% for already-stable Northeast cities. This implies the depreciation can easily over take the effect of a growing city's border, especially once the activity circle is completely developed.

Concluding Remarks

The relationship between the growth rate of a city and the apartment rents has been long argued, but little empirical evidences have been proposed. This study provides the empirical evidence by pointing out that the growth rate and the rents are positively related to each other. Apartments in faster growing areas rent at higher rates than those in slower growing areas.

Although the standard urban economic theory is shown to fit the data, it is also shown that the effect is only a part of the dramatic rent hike. To fulfill the gap, the concept of the "activity circle" is introduced. This concept is meant to overcome some of major problems of the standard theory, such as ignoring consumer utilities from varieties.

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[27] These results are from estimating regressions without location factors or the growth rate of the cities. Slower depreciation for the South and faster depreciation for the Northeast suggests that the growth rate of

An “activity circle” proxies the number of varieties of daily activities available in a resident’s immediate neighborhood and the utility gain from it. The utility gain is the amount the resident is willing to pay additional to the rent the standard model estimates. The empirical evidence shows this “activity circle” combined with the standard theory provides a better fitting model to the Florida apartment data than the standard theory only[28].

There are some characteristics of the Florida apartment market that might cause a need for some caution in interpreting the results of this study. First, there was a boom in apartment construction from 1983 through 1987 due to accelerated depreciation and provisions for the Economic Recovery and Tax Act of 1981, which was removed in 1986. This apartment construction boom might have increased competitions, especially non-price competitions, both among those constructed newly and between newly constructed and existing complexes. The non-price competition could be reflected in the observable amenities, but there is no apparent difference between those built during the boom and the other apartments[29]. However, if we look at the time those apartments were built, there are slight differences in amenities. For example, more apartments built during the boom have a fitness center or provide a microwave oven in the unit than those built earlier (the second and the fifth columns in Table B-2). It seems the apartments built during the time the competition was intense provide more amenities (higher quality?)

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a city might perform an important role in the rent depreciation process. Thus, Lane, Randolph and Berenson may provide some additional implicit evidence in favor of the C-H model.

[28] As it is mentioned earlier, the apartment rent data shows the effect of the activity circle more clearly than housing prices. House prices reflect the entire present discounted value of the flow of the value of the future use, including the value of the variety in the local neighborhood, minus maintenance costs, property tax and so on. To show the difference between the housing price and rental rates, figure B-1 in Appendix B compares the house price index from the State of Florida’s Housing (1998) to the rent index I have estimated.

than the existing complexes at that time, but after the boom was over, newly constructed apartments had to have more or less the same amenities (quality) to compete against them. Also, having more luxurious amenities has been the trend movement in recent years. These differences in the level of quality of apartments caused by changes in the external events, such as changes in the laws, must be controlled by the characteristic variables within the hedonic rent function. Having well-specified characteristic variables and the variable MOTEL in the model is very advantageous to overcome the problem associated with such time-specific characteristics.

Second, the Florida's population growth has slowed sharply starting in 1990. This is a result of recession and trend changes. The Great Depreciation lowered the birth rate in 1930s, causing fewer people to turn 60 and retire to Florida in 1990s. Furthermore, the baby boomers started entering less mobile ages, and that brings fewer people to Florida. The empirical study with the C-H model in the last part of this study uses the population growth rate estimated for the 1980-1990 period, so the C-H model alone might estimate a even lower rent increase with more current growth rate data. That means the "activity circle" effect might be more significant than the empirical study in this paper implies.

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[29] See table B-2 for comparisons of the amenities of apartments built during the boom and those built in other time.

CHAPTER 5
CONCLUSION AND SUMMARY

Throughout this dissertation, it is demonstrated that overlooking the effect of changes in varieties of goods and services available for consumers might result in a misleading analysis. Consumers value the benefit from having more choices significantly higher than previous studies suggest.

In chapters two and three, consumers are introduced to new varieties of commodities through imports. Restricting the opportunity for consumers to widen their choice bundle by imposing tariffs results in a great reduction in the consumers' welfare. In chapter four, the consumers' choice bundle is widened by the development of an activity circle within their neighborhood. The empirical research in this chapter concludes the dollar value of consumers' additional utility gain from increased variety is reflected in the rent they pay to live in a particular location.

Consumers' utility and welfare is difficult to measure, but changes in them can be indirectly observed. There are numerous theoretical analyses of consumers' utility and welfare in many fields, but little attention has been paid to provide empirical evidence for the theories. Because the accuracy and details of available data and econometrics techniques are improved dramatically in the last few decades, it is the time to connect the theoretical models to the real world situations.

APPENDIX A
APPENDIX FOR CHAPTER THREE

Relationship between N and P

The relationship is illustrated by simulating the derived conditions for the equilibrium price and the equilibrium number of firms. Assume the following values for the parameters.

$\alpha = 100$
$\beta = 0.5$
$c = 1$
$r = 0.1$
$\mu = 80$
$\upsilon = 30$

Table A-1 shows the numerical values for the two variables when the tariff rate is reduced. This relationship is illustrated in Figure 3-1 in the text.

Table A-1. Numerical Result

Tariff	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
N	1.069	1.384	1.599	1.767	1.908	2.031	2.141	2.240	2.332
P	70.00	68.33	67.78	67.50	67.33	67.22	67.14	67.08	67.04

Consumer Surplus and Tariff Revenue

The difference in the consumer surplus in the two periods is

$$CS_2 - CS_0 = \frac{2}{27\beta}\left(\alpha - \frac{c}{1-\tau}\right)\left[2\beta(1-\tau)\right]^{1/3}\left[\frac{1}{N_2^{3/2}\upsilon^{1/3}} - \frac{1}{N_0^{3/2}(r\mu+\upsilon)^{1/3}}\right]. \qquad \text{(A-1)}$$

To determine the sign of the last brackets, consider the magnitudes of each denominator:

$N_2^{3/2}\upsilon^{1/3}$ and $N_0^{3/2}(r\mu+\upsilon)^{1/3}$.

$$N_2^{3/2}\upsilon^{1/3} = \left[\frac{1}{3\beta}\left(\alpha-\frac{c}{1-\tau}\right)\right]^{3/4}\left[\frac{2\beta(1-\tau)}{\upsilon^{1/3}}\right]^{1/2}$$

$$> \left[\frac{1}{3\beta}\left(\alpha-\frac{c}{1-\tau}\right)\right]^{3/4}\left[\frac{2\beta(1-\tau)}{(r\mu+\upsilon)^{1/3}}\right]^{1/2} = N_0^{3/2}(r\mu+\upsilon)^{1/3}. \qquad \text{(A-2)}$$

Therefore, the consumer surplus created by each variety in the period 2 is smaller than the consumer surplus in the initial period. Similarly the difference in the tariff revenue is

$$TR_2 - TR_0 = \tau\alpha\left[\frac{1}{3\beta}\left(\alpha-\frac{c}{1-\tau}\right)\right]^{1/2}\left[\frac{1}{N_2^{1/2}\upsilon^{1/3}}-\frac{1}{N_0^{1/2}(r\mu+\upsilon)^{1/3}}\right]$$

$$-\frac{\tau}{9\beta}\left(\alpha-\frac{c}{1-\tau}\right)^2\left[2\beta(1-\tau)\right]^{1/3}\left[\frac{N_2^{1/2}}{\upsilon^{1/3}}-\frac{N_0^{1/2}}{(r\mu+\upsilon)^{1/3}}\right]. \qquad \text{(A-3)}$$

Working similarly to the previous case, it can be easily shown that $N_2^{1/2}\upsilon^{1/3} > N_0^{1/2}(r\mu+\upsilon)^{1/3}$ and $\frac{N_2^{1/2}}{\upsilon^{1/3}} > \frac{N_0^{1/2}}{(r\mu+\upsilon)^{1/3}}$, so that the tariff revenue from one variety is lower in period 2 than in the initial period.

APPENDIX B
APPENDIX FOR CHAPTER FOUR

Data Set

The data set used in this study is collected by the Bureau of Economic and Business Research for the Florida Price of Living program. The rent data contains 3,579 samples of apartment complexes from 61 counties in Florida. Apparently, there are some counties without any registered complexes. Each apartment complex is interviewed over the phone during August and September of 1997 for over 40 questions specifying structural characteristics and neighborhood qualities of the complex. Then the straight line distances from the nearest activity centers are measured as proxies for the accessibility to them. Complexes which receive government subsidies are dropped from the data because it adds some complexity of difference between the rent and the real rent. the data is then reshaped into a panel data, not by time and observation like ordinary panel data is, but by complexes and number of bedrooms. In this way, the value of one additional bedroom can be more clearly measured while controlling the shared amenities.

One potential danger of collecting data by interviewing is inclusion of some subjectivity of the interviewees. To minimize the danger, the data set collected in 1997 is compared to the one collected in the previous year. Observations are employed only if those two sets of data match reasonably well.

Estimation Results for Hedonic Rent Function

Table B-1: Hedonic Estimation Results

Variable	Coeff.	t-stat.	Variable	Coeff.	t-stat.
LUNITS	-.0010646	-0.239	GOLF5	-.0227111	-2.520
GATE	.0119636	0.916	GOLFFAR	-.0483145	-4.265
FURN	.0410597	2.550	COA0	.1165023	5.975
COVERED	.0466427	4.848	COA1	.1193385	8.319
GARAGE	.1192348	9.888	COA2	.0643539	5.691
CABLE	.0145282	1.807	COA3	.0356471	3.009
DISH	.0343243	4.221	COA7	-.0011055	-0.103
STORAGE	.0064037	1.275	COA10	-.0385699	-3.718
OSMNG	.0212846	2.081	COA15	-.0455151	-4.084
EMG24	.0164679	1.054	COA20	.0127275	0.657
POOL	.0271667	2.974	COA30	-.0558556	-3.548
FITCTR	.0560014	8.194	COAFAR	-.0524389	-2.020
CLBHOUSE	-.0036198	-0.597	SCH0	-.0059333	-.765
TENNIS	.0048027	0.724	SCH1	.004875	0.672
PET	.045992	7.444	SCH2	.0191298	1.605
MWAVES	.0525492	6.805	SCH3	.0368826	2.507
WD	.039043	5.819	SCH4	.0024433	0.093
PATIO	.0352676	5.255	SCHFAR	.1066566	3.084
SIZE	.2444838	13.640	AGE4	-.1468507	-2.876
MOTEL	.1209037	8.363	AGE6	-.1393611	-2.700
BATH	.0484672	6.189	AGE8	-.1007734	-1.990
BEDR	.0987211	16.341	AGE10	-.0347626	-1.283
UPSCALE	.060238	10.175	AGE12	-.0836831	-1.657
CITY1	-.0082446	-0.778	AGE14	-.0933622	-1.837
CITY2	.0040134	0.488	AGE16	-.0531967	-1.036
CITY7	-.0236403	-3.283	AGE18	-.1202049	-2.316
CITY10	-.0087861	-0.892	AGE20	-.1772772	-3.435
CITY15	-.0182942	-1.501	AGE22	-.1863724	-3.607
CITY20	-.0136667	-0.830	AGE24	-.1716438	-3.343
CITY30	.0029409	0.124	AGE26	-.1679379	-3.302
CITY50	-.0570391	-2.295	AGE30	-.150538	-2.960
CITYFAR	.1038692	1.466	AGE40	-.1625577	-3.223
GOLF0	.0035569	0.318	AGE50	-.1776988	-3.238
GOLF2	-.0095453	-1.256	AGEOLD	-.0824214	-1.518
GOLF3	.0053514	0.630			

Table B-1—continued

Variable	Coeff.	t-stat.	Variable	Coeff.	t-stat
Alachua	-.1710186	-5.795	Lee	-.2066712	-11.014
Baker	.1231316	1.375	Leon	-.1845151	-8.674
Bay	-.3988822	-5.264	Levy	dropped	
Bradford	dropped		Liberty	dropped	
Brevard	-.2915546	-16.610	Madison	-.3193949	-3.519
Broward	.0102844	0.862	Manatee	-.1853108	-8.513
Calhoun	-.0896707	-1.121	Marion	-.3190667	-8.995
Charlotte	-.6906719	-8.843	Martin	-.260573	-4.852
Citrus	-.4939537	-10.136	Monroe	-.1745897	-2.093
Clay	-.160029	-5.989	Nassau	-.2920705	-6.169
Collier	-.1246085	-3.453	Okaloosa	-.2138955	-6.330
Columbia	dropped		Okeechobee	-.4325816	-4.779
De Soto	-.3407757	-5.387	Orange	-.1379791	-5.055
Dixie	-.3545546	-3.987	Osceola	-.1657237	-4.961
Duval	-.1808735	-13.640	Palm Beach	-.0625902	-4.385
Escambia	-.2516911	-12.780	Pasco	-.3320333	-12.631
Flagler	dropped		Pinellas	-.2798598	-20.657
Franklin	dropped		Polk	-.235416	-9.568
Gadsden	dropped		Putnam	-.3149529	-6.870
Gilchrist	-.8607429	-9.294	St. Johns	-.1925652	-4.725
Glades	-.358969	-3.112	St. Lucie	-.2995715	-7.317
Gulf	dropped		Santa Rosa	-.390908	-8.755
Hamilton	dropped		Sarasota	-.283802	-5.485
Hardee	-.5758225	-7.190	Seminole	-.144572	-5.178
Hendry	dropped	.	Sumter	-.7138659	-6.634
Hernando	-.4461718	-8.367	Suwannee	-.8455035	-7.618
Highlands	-.5084502	-9.771	Taylor	-.1179264	-.-1.797
Hillsborough	-.2489002	-22.003	Union		
Holmes	dropped		Volusia	-.3042536	-18.141
Indian River	-.4118432	-8.799	Wakulla		
Jackson	-.3693564	-5.148	Walton	-.3714858	-4.338
Jefferson	-.1297769	-1.442	Washington		
Lafauette	dropped		CONSTANT	4.566523	36.744
Lake	-.2795374	-8.108			

$R^2 = 0.7857$

Proof for A' and A''

Define $z_y = b_0\left(\dot{b}-1\right)$: the city's radius increased since time t_0. Then the city area with in an individual's activity circle given by equation (4-8) becomes

$$A_t = \frac{r^2\pi}{2} + r^2 \sin^{-1}\left(\frac{z_t}{r}\right) + z_t(r^2 - z_t^2)^{1/2}. \qquad \text{(B-1)}$$

Differentiate it with respect to *t*.

$$A_t' = \frac{dA_t}{dt} = \left[r^2 \frac{1}{r\sqrt{r^2 - z_t^2}} + \left(r^2 - z_t^2\right)^{1/2} - z_t^2\left(r^2 - z_t^2\right)^{-1/2} \right] \frac{dz_t}{dt}$$

$$= 2(r^2 - z_t^2)^{1/2} \frac{dz_t}{dt}, \qquad \text{(B-2)}$$

where $\frac{dz_t}{dt} = b_0\dot{b}^t \ln b > 0$. Therefore, A_t' is positive (or zero when the entire activity circle is occupied by the city area) for all *t*.

The second derivative of function A_t is

$$A_t'' = \frac{d^2A_t}{dt^2} = -2z_t(r^2 - z_t^2)^{-1/2} \frac{d^2z_t}{dt^2}, \qquad \text{(B-3)}$$

where $\frac{d^2z_t}{dt^2} = b_0\dot{b}^t(\ln b)^2 > 0$. Thus A_t'' is negative (or zero when the entire activity circle becomes the city area) for all *t*.

Effect of the Apartment Construction Boom in Florida

Table B-2. Comparison of Amenities

	Apartments built between 1983 and 1987		Apartments built in other than 1983-1987		Apartments built Before 1983	
Variables	Mean	Standard Error	Mean	Standard Error	Mean	Standard Error
UNITS	226.03	142.313	214.373	148.578	177.274	130.921
GATED	0.19587	0.39708	0.24576	0.43063	0.18891	0.39156
FURN	0.01088	0.10380	0.02758	0.16383	0.03482	0.18337
COVERED	0.7399	0.26190	0.09295	0.29043	0.06770	0.25131
GARAGE	0.01959	0.13865	0.07980	0.27104	0.03095	0.17323
CABLE	0.15343	0.36060	0.10187	0.30254	0.08188	0.27427
DISH	0.82590	0.37970	0.80645	0.39516	0.73114	0.44351
STORAGE	0.45158	0.49792	0.35823	0.47958	0.29529	0.45632
OSMNG	0.9456	0.22694	0.94397	0.23002	0.93617	0.24453
EMG24	0.99347	0.08058	0.97326	0.16136	0.96905	0.17323
POOL	0.82372	0.38126	0.88243	0.32217	0.84397	0.36300
FITCTR	0.53120	0.49924	0.43930	0.49641	0.25210	0.43436
CLBHOUS	0.62677	0.48393	0.63073	0.48271	0.52547	0.49951
TENNIS	0.6638	0.47268	0.54117	0.49841	0.42862	0.49478
PET	0.90751	0.28988	0.77207	0.41959	0.72405	0.44714
MWAVES	0.20022	0.4004	0.13795	0.34492	0.04126	0.19896
WD	0.30686	0.46144	0.23132	0.42177	0.07995	0.27130
PATIO	0.9456	0.22694	0.83659	0.36982	0.80658	0.39511
SIZE	954.46	221.780	972.89	238.33	949.73	245.18
MOTEL	0.10530	0.22630	0.06093	0.23529	-0.02878	0.19391
BATH	1.5539	0.48629	1.50573	0.48234	1.44742	0.47328
BEDR	1.7171	0.66866	1.79207	0.73562	1.75306	0.72576
RENT	656.20	175.930	641.523	218.676	570.236	169.922
CITY[1]	8.7359	12.6314	6.52610	8.33634	6.4867	8.58927
COAST[1]	14.384	14.8514	17.0227	16.0995	16.4053	15.5163
GOLF[1]	2.4580	1.99158	2.41188	1.87155	2.53125	1.98554
SCHOOL[1]	0.9056	0.63715	0.82408	0.69142	0.75449	0.62085
UPSCALE	0.47225	0.49950	0.42699	0.49475	0.29336	0.45545
Observation	919		2356		1551	

[1] Distances are in mileage.

House Price vs. Rent

The C-H model and the activity circle are both applicable to the single family housing market as well as the apartment market, but testing the model is easier with apartment rent data. A house price is the present value of the flow of *all* future use, not the value of the services provided in a certain period. In particular, the price of a house includes the flow of expected maintenance costs, property taxes, and the value of the land as a capital asset. In addition, all the future effects of the activity circle as well as the city growth are capitalized in the price of a house. These factors complicate the analysis of the effect of the activity circle on the house price.

A comparison between the single-family housing price and the apartment rent is can show the difference. The single-family housing price is a repeat-sale house price index taken from the State of Florida's Housing (1998) for 1996 sales price with the base of 1980 sales price adjusted for inflation. Because the house price index is given for nineteen Metropolitan Statistical Areas (MSAs) in Florida, the rent index also is estimated for the same MSAs. The rent index is compared to the house price as follows. Because the repeat-sale house price index does not control for the age of houses, the average age of apartments in each MSA is calculated. Then the rent estimated by a hedonic function for an apartment of that age is compared to the estimated rent for an apartment that constructed sixteen years earlier[30]. The hedonic rent function use to estimate the rent function has the following functional form.

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[30] Notice that the house price index is the ratio of the price in 1996 to the price in 1980. There is a sixteen-year gap between the two observations. The estimated hedonic rent index must have the same time gap.

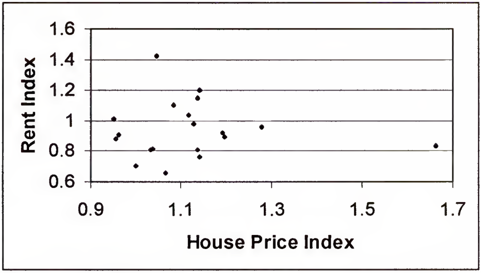

Figure B-1. Comparison between House Price Index and Rent Index

$$R_i = \beta_0 + \sum_{j=1}^{n} \beta_{1j} X_{ji} + \sum_{k=1}^{m} \beta_{2k} Y_{ki} + \sum_{l=1}^{v} \sum_{r=1}^{z} \beta_{3lr} D_{lri} + \sum_{s=1}^{p} \sum_{u=1}^{q} \beta_{4su} A_{si} M_{ui} + \sum_{w=1}^{q} \beta_{6w} M_{wi} , \quad \text{(B-4)}$$

where M is a set of MSA dummy variables.

Figure B-1 shows the result of the comparison for the nineteen MSAs. There is no apparent relationship between the house price and the apartment rent. That is, it is possible to find an apartment whose rent increase is slow while housing prices rise very quickly in the same MSA and vice versa.

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BIOGRAPHICAL SKETCH

Kayoko Ishikawa entered the Southern Illinois University at Carbondale in Niigata, Japan in 1989 following a period of intensive English study. After two years of a general education program, she transferred to the Southern Illinois University at Carbondale, USA to complete the degree of Bachelor of Art in Economics. She earned the degree in May 1993 with honors. During the two years in SIUC, she received the awards for the best third year student in the Department of Economics, for the best forth year student in the Department of Economics, and for the best foreign student.

Kayoko Ishikawa then enrolled in the Ph.D. program at the Department of Economics at the University of Florida. After completing 16 core courses in two years, she passed the written qualify exams in the fields of international economics and econometrics.

While she was completing her study as a graduate student, she has worked as a teaching assistant and a research assistant. She has been a teaching assistant assisting professors in introductory microeconomics and macroeconomics courses and international economics courses for undergraduate and MBA students. She has taught two courses in managerial economics. She also has worked as a research assistant for the Bureau of Economic and Business Research assisting in the construction of a price index for apartment rents in a part of Florida Price of Living Index program.

She is expecting to receive her Ph.D. in May 1998.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts.

Mark Rush

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Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts.

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This thesis was submitted to the Graduate Faculty of the Department of Geology in the College of Liberal Arts and Sciences and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Master of Arts.

May, 1998

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